# 國立中央大學九十一學年度轉學生入學試題卷



1. 兩火車站間有一列火車的速度如下：

$$v(t)=\begin{cases} a(t^3-3t^2) & 0\le t<2 \\ 4 & 2\le t<4 \\ b(2t^3-27t^2+120t-175) & 4\le t<5 \end{cases}$$

（$v$ 為火車速度；$t$ 為時間）。如果火車的速度是連續的，則 $a$、$b$ 的值為何？（5%）

2. 令函數 $f(x)=|x|$，請問 (i) 該函數的導函數為何？（5%） (ii) 該函數在 $x=0$ 處，是否連續？又是否可微？（5%）

3. 求 (i) $\frac{d}{dx}\ln(\frac{x+a}{x-a})$ （5%） (ii) $\frac{d}{dx}\sin^{-1}x$ （5%）

4. 令 $f(x)=yz/x$，求 $f_x$，$f_y$，$f_z$ 及 $df$。（10%）

5. 求 (i) $\int_0^{\pi/4}\sin\theta\ln(\cos\theta)d\theta$ （5%） (ii) $\int_0^1\int_{y^2}^{y}2x(1+xy)dxdy$ （5%）

6. 令三維空間向量 $\vec{A}=(4xz,-y^2,yz)$，並且 $V$ 為封閉曲面 $S$: $x=0, x=1, y=0, y=1, z=0, z=1$ 所包圍的單位立方體。(i) 求 $\nabla\bullet\vec{A}$ （5%） (ii) 求體積分 $\int_V\nabla\bullet\vec{A}dV$ （5%） (iii) 求面積分 $\oint_S\vec{A}\bullet d\vec{S}$ （5%） (iv) 請簡單陳述高斯發散定理（Gauss's divergence theorem）（5%）。

7. Within the spatial interval $(0,L)$, please derive a solution formula of the one-dimensional heat equation

$$\frac{\partial u(x,t)}{\partial t}=c^2\frac{\partial^2 u(x,t)}{\partial x^2} \qquad \text{(Eq. 1)}$$

with two Neumann's boundary conditions: $u_x(0,t)=0$ and $u_x(L,t)=0$ for all $t$

and one initial condition: $u(x,0)=f(x)$. （10%）

參考用

注意：背面有試題

8. Considering a one-dimensional motion $x(t)$ of a particle is governed by the following nonlinear differential equation:

$$\dot{x} = \sin x \qquad \text{(Eq. 2)}$$

where $x$ is the position of this particle and $\dot{x}$ is the time-derivative of its position, i.e. the velocity.

(i) Suppose that $x(t=0) = x_0$, please find the position function $x(t)$ for this particle. (5%) (Hint: $\int \csc u\,du = -\ln|\csc u + \cot u| + C$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$)

(ii) Assuming $x_0 = \pi/4$, please plot the solution $x(t)$ qualitatively. What happens as $t \to \infty$? (5%)

(iii) Now let consider another thinking way for (Eq. 2). Please plot (Eq. 2) in the so-called phase plane, that is the position as the abscissa and the velocity as the ordinate or the $x$-$\dot{x}$ plane, and simply by arrows indicate the moving directions of our imaginary particle everywhere. (5%)

(iv) For an arbitrary initial condition $x_0$, what is the behavior of $x(t)$ as $t \to \infty$? (5%) (Hint: Consider those points $x = n\pi$, $n$ is an integer)

(v) For any one-dimensional system $\dot{x} = f(x)$ as shown in the figure below, could you describe the general behavior of the particle motion qualitatively as the evolution of time? (5%)