

國立中央大學九十一年度轉學生入學試題卷

地球科學學系 三年級

科目：應用數學

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1. 兩火車站間有一列火車的速度如下：

$$v(t) = \begin{cases} a(t^3 - 3t^2) & 0 \leq t < 2 \\ 4 & 2 \leq t < 4 \\ b(2t^3 - 27t^2 + 120t - 175) & 4 \leq t < 5 \end{cases}$$

(v 為火車速度； t 為時間)。如果火車的速度是連續的，則 a 、 b 的值為何？(5%)

2. 令函數 $f(x) = |x|$ ，請問 (i) 該函數的導函數為何？(5%) (ii) 該函數在 $x = 0$ 處，是否連續？又是否可微？(5%)

3. 求 (i) $\frac{d}{dx} \ln\left(\frac{x+a}{x-a}\right)$ (5%) (ii) $\frac{d}{dx} \sin^{-1} x$ (5%)

4. 令 $f(x) = yz/x$ ，求 f_x 、 f_y 、 f_z 及 df 。(10%)

5. 求 (i) $\int_0^{\pi/4} \sin\theta \ln(\cos\theta) d\theta$ (5%) (ii) $\int_0^1 \int_0^y 2x(1+xy) dx dy$ (5%)

6. 令三維空間向量 $\vec{A} = (4xz, -y^2, yz)$ ，並且 V 為封閉曲面 $S: x=0, x=1, y=0, y=1, z=0, z=1$ 所包圍的單位立方體。(i) 求 $\nabla \cdot \vec{A}$ (5%) (ii) 求體積分 $\int_V \nabla \cdot \vec{A} dV$ (5%) (iii) 求面積分 $\oint_S \vec{A} \cdot d\vec{S}$ (5%) (iv) 請簡單陳述高斯發散定理 (Gauss's divergence theorem) (5%)。

7. Within the spatial interval $(0, L)$, please derive a solution formula of the one-dimensional heat equation

$$\frac{\partial u(x, t)}{\partial t} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2} \quad (\text{Eq. 1})$$

with two Neumann's boundary conditions: $u_x(0, t) = 0$ and $u_x(L, t) = 0$ for all t

and one initial condition: $u(x, 0) = f(x)$. (10%)

參考用

注意：背面有試題

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8. Considering a one-dimensional motion $x(t)$ of a particle is governed by the following nonlinear differential equation:

$$\dot{x} = \sin x \quad (\text{Eq. 2})$$

where x is the position of this particle and \dot{x} is the time-derivative of its position, i.e. the velocity.

- (i) Suppose that $x(t=0) = x_0$, please find the position function $x(t)$ for this particle.

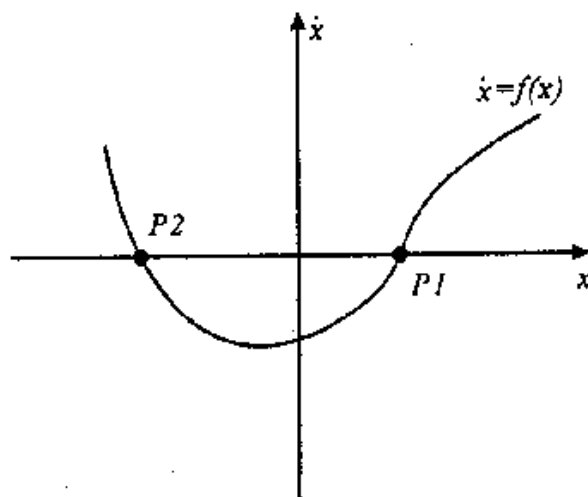
(5%) (Hint: $\int \csc u du = -\ln|\csc u + \cot u| + C$ and $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$)

- (ii) Assuming $x_0 = \pi/4$, please plot the solution $x(t)$ qualitatively. What happens as $t \rightarrow \infty$? (5%)

- (iii) Now let consider another thinking way for (Eq. 2). Please plot (Eq. 2) in the so-called phase plane, that is the position as the abscissa and the velocity as the ordinate or the $x-\dot{x}$ plane, and simply by arrows indicate the moving directions of our imaginary particle everywhere. (5%)

- (iv) For an arbitrary initial condition x_0 , what is the behavior of $x(t)$ as $t \rightarrow \infty$? (5%)
(Hint: Consider those points $x = n\pi$, n is an integer)

- (v) For any one-dimensional system $\dot{x} = f(x)$ as shown in the figure below, could you describe the general behavior of the particle motion qualitatively as the evolution of time? (5%)



參考用