

國立中央大學八十六學年度轉學生入學試題卷

三年級

科目:

數量方法

共二頁 第

I) Calculus (50%)

- 1) (5%) Find the total differential of the function:
 a) $y = 3x_1^2 + x_1x_2^2$
 b) $y = (x_1 + x_2)/2x_1^2$
- 2) (5%) Prove $\log_b u = (\log_e e)(\log_e u)$.
- 3) (5%) Find the Taylor series, with $n=4$ and $x_0=-2$, for the function $\phi(x) = (1-x)/(1+x)$.
- 4) (10%) Suppose the value of timber (already planted on some given land) is the following increasing function of time:
 $v = 2^{2t}$
 expressed in units of \$1000. Assuming a discount rate of r (on the continuous basis) and also assuming zero upkeep cost during the period of timber growth, what is the optimal time to cut the timber for sale?
- 5) (5%) Find the extreme value(s) of $z = x + 2ey - e^x - e^{2y}$, and determine whether they are maxima or minima.
- 6) (5%) Evaluate the following functions:
 a) $\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{x - 4}$
 b) $\lim_{x \rightarrow 0} \frac{5^x - e^x}{x}$
- 7) (10%) Find a) $\int xe^x dx$
 b) $\int (5e^x - x^2 + (3/x)) dx$.
- 8) (5%) Evaluate a) $\int_1^2 e^{2x} dx$
 b) $\int_2^3 (e^{2x} + e^x) dx$.

II) Statistics (50%)

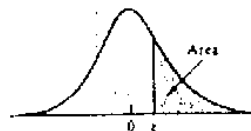
- 1) (5%) If a card is drawn from a deck of playing cards, what is the probability that it will be a jack or a ten?
- 2) (5%) What is the probability of getting exactly 3 heads in 5 flips of a balanced coin?
- 3) (5%) Use Chebyshev's inequality to find a lower bound on $\Pr(-4 < x < 20)$ where the random variable x has a mean $\mu = 8$ and variance $\sigma^2 = 9$.
- 4) (5%) If x follow a discrete uniform distribution, i.e. $F(x) = 1/N$ for $x = 1, 2, \dots, N$. Find $E(x)$ and $\text{Var}(x)$.
- 5) (5%) An electrical firm manufactures light bulbs that have a lifetime that is normally distributed with a mean of 800 hours and a standard deviation of 40 hours. If 100 bulbs, about how many will have lifetimes between 778 and 834?
- 6) (5%) A research worker wishes to estimate the mean of a population using a sample large enough that the probability will be 0.95 that the sample mean will not differ from the population mean by more than 25 percent of the standard deviation. How large a sample should he take?

(10%) The student government of a large college polled a random sample of 325 male students and found that 221 were in favor of a new grading system. At the same time, 120 out of a random sample of 200 female students were in favor of the new system. Do the results indicate a significant difference in the proportion of male and female students who favor the new system? Test at the 0.05 level of significance.

(10%) An old farmer had bunions on his feet. He noticed that there seemed to be a relation between the number of business and inches of rainfall on his farm. He collected data for a week and then computed the following summary statistics. If X = number of bunions and Y = inches of rainfall, the farmer found that $\Sigma X/7 = 4$, $\Sigma Y/7 = 4$, $\Sigma XY = 94$, $\Sigma X^2 = 124$. If the farmer wakes up with 5 bunions, how many inches of rain do you predict that day?

Tables

TABLE 4 Normal curve areas
Standard normal probability in right-hand tail
(for negative values of z areas are found by symmetry)



z	Second decimal place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000233									
4.0	.0000317									
4.5	.00000340									
5.0	.000000287									