國立中央大學八十八學年度轉學生入學試題卷

財務管理學系　三年級　　科目：統計學　　　　共 1 頁　第 1 頁

Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, say, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

1. (50 points) Suppose there are 4 boxes, labeled 1, 2, ..., 4. Five balls, labeled 1, 2, ..., 5, are distributed at random to these boxes (Note: there will be one ball left). Let X_i denote the number of the ball contained in box i. Also, let S_i denote a random variable which equals 1 if the number of the ball contained in the box i is also i, and zero otherwise.
   (a) (10 poinst) Calculate the mean and variance of X_i.
   (b) (10 poinst) Calculate the mean and variance of S_i.
   (c) (10 points) Calculate the correlation of X_i and X_j, i.e., $corr(X_i, X_j)$.
   (d) (10 points) Calculate the correlation of S_i and S_j, i.e., $corr(S_i, S_j)$.
   (e) (10 points) Calculate the probability that $S_1 + ... + S_4 = 2$.
2. (20 points) Suppose you are testing $H_0 : p = 1/2$ against $H_1 : p = 1$ for a binomial variable with $n = 2$. List all critical regions for which the type I error $\alpha \leq 1/2$. Which of these ciritical regions minimizes the sum of type I and type II errors $(\alpha + \beta)$?
3. (10 points) Suppose a random variable z is known to have a chi-square (χ^2) distribution with ν degrees of freedom, and $w = 2z$. Calculate $E(w)$ and $Var(w)$. What do you know about the distribution of w?
4. (20 points) Let r_t, $t = 1, ..., T$, denote an *iid* random varaible with a normal distribution whose mean is μ and variance is 1.
   (a) (10 points) Calculate the mean and variance of the number of r_t's that are greater than 0. (you may express your answer in terms of normal cdf.)
   (b) (10 points) Derive a test statistic to test the null hypothesis that the mean μ is zero, i.e., $H_0 : \mu = 0$.