國立中央大學八十九學年度轉學生入學試題卷

財務管理學系　三年級　　科目：統計學　　　　共 1 頁　第 1 頁

Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, say, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0, 1)$.

1. (30%) There are three boxes. Box A contains 2 white balls and 2 black balls, box B contains 2 white balls and 1 black ball, and box C contains 1 white ball and 3 black balls.

   (a) A ball is selected from each box. Calculate the probability of getting all white balls.

   (b) One box is selected at random and one ball drawn from it. Calculate the probability that it will be white.

   (c) In (b), calculate the probability that the first box was selected given that a white ball is drawn.

2. (20 %) Suppose n balls are distributed at random into r balls. Let $X_i = 1$ if box i is empty and let $X_i = 0$ otherwise.

   (a) Compute $E(X_i)$.

   (b) For $i \neq j$, compute $E(X_i X_j)$.

   (c) Let S_r denote the number of empty boxes. Write $S_r = X_1 + \ldots + X_r$. Compute $E(S_r)$.

3. (20 %) Let r_t, $t = 1, \ldots, T$, denote an *iid* random variable with a normal distribution whose mean is μ and variance is 1. Derive a test statistic to test the null hypothesis that the mean μ is zero, i.e., $H_0 : \mu = 0$. Write down the statistic and its distribution explicitly.

4. (30 %) If the stock price p_t follows an *iid* random walk process as follows:

$$p_t = 1 + p_{t-1} - 3e_t, t = 1, \ldots, T;$$

   where e_t is continuously uniformly distributed between 0 and 1, i.e., $e_t \overset{iid}{\sim} \mathcal{U}(0, 1)$. Let $r_t = p_t - p_{t-1}$ be the stock return. Define the following random variable:

$$I_t = \begin{cases} 1 & \text{if } r_t \geq 0 \\ 0 & \text{otherwise.} \end{cases} \qquad (1)$$

   Also define the following two random variables:

$$\begin{aligned} Y_t &= I_t I_{t+1} + (1 - I_t)(1 - I_{t+1}) \\ N &= \sum_{t=1}^{T} Y_t \end{aligned}$$

   Answer the following questions:

   (a) Calculate the variance of Y_t, $Var(Y_t)$ (Hint: Tabulate all possible outcomes of Y_t's and their probabilities).

   (b) Calculate the variance of N, $Var(N)$.