Linear Algebra

Part 1. (10 points each)

Let V be a subspace of a real n-dimensional vector space R^n. The n by n matrix A is the projection matrix onto V if

a. $A \cdot x = x$ if $x \in V$,

b. $A \cdot y = 0$ if y is in the orthogonal complement of V.

[1] Prove or disprove that $A^2 = A$ and $A^t = A$ if and only if A is a projection matrix.

[2] Let V be a subspace spanned by (1,1,1,1),(1,1,-1,-1), (1, 1,-1,1) . Find the projection matrix.

[3] Let V be the subspace defined in [2]. Find the projection of the vector b=(1,2,3,4) onto V.

[4] Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$. Find a vector x in R^3 such that the norm $\| A \cdot x - b \|$ is minimal.

[5] Let $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Find y in R^4 such that $A \cdot y = b$ and the norm $\| y \|$ is minimal

Part 2.

[1] Prove or disprove that a complex square matrix A has only real eigen values if and only if A is a Hermitian matrix. (20 points)

[2] Let $A = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$

[a] Find the L-U decomposition of A. (15 points)

[b] Prove or disprove that A is positive definite. (15 points)