

國立中央大學八十九學年度轉學生入學試題卷

15 數學系 三年級

科目：線性代數

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1. Let A be a real n by n matrix. Prove or disprove that the sum of the n eigenvalues of matrix A equals the sum of the n diagonal entries of A . (20%)
2. Let A and B be real n by n matrices. Prove or disprove that $\text{Trace}(AB) = \text{Trace}(BA)$. (Note. The trace (A) equals the sum of the n diagonal entries of A .) (20%)
3. Let V and W be subspaces of \mathbb{R}^n with $V \subseteq W \subseteq \mathbb{R}^n$. Prove that the orthogonal complements V^\perp and W^\perp have the relation $V^\perp \supseteq W^\perp$. (20%)
4. Let A be a real m by n matrix. Prove or disprove that the subspace $\{x \mid Ax = 0\}$ of \mathbb{R}^n equals the subspace $\{x \mid A^T A x = 0\}$. (Note. The matrix A^T is the transpose of A .) (20%)
5. Find a diagonal matrix Λ and an invertible matrix S such that
$$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} = S \Lambda S^{-1}. \quad (20\%)$$