國立中央大學97學年度碩士班考試入學試題卷

所別：地球物理研究所碩士班 一般生　科目：應用數學　共 1 頁 第 1 頁

學位在職生

＊請在試卷答案卷（卡）內作答

1. Solve the nonhomogeneous system,

$$\begin{cases} y_1' = 4y_1 - 2y_2 + t^{-3} \\ y_2' = 8y_1 - 4y_2 - t^2 \end{cases}, \quad t > 0 \qquad [20\%]$$

2. (a) What is the Cauchy principal value of an improper integral $\int_A^B f(x)dx$ whose integrant $f(x)$ becomes infinite at a point a, $a \in (A,B)$. [15%]

(b) Evaluate (show details) the Cauchy principal value of

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x-2)} \qquad [15\%]$$

3. (a) Write down the three fundamental partial differential equations in physics and engineering: (i) Laplace equation; (ii) heat equation; (iii) wave equation, in arbitrary coordinate systems. [15%]

(b) In spherical coordinates, the Laplacian operator ∇^2 can be written as:

$$\nabla^2 = \frac{1}{r^2}\left[\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{\sin\phi}\frac{\partial}{\partial\phi}(\sin\phi\frac{\partial}{\partial\phi}) + \frac{1}{\sin^2\phi}\frac{\partial^2}{\partial\theta^2}\right]$$

Using separating variables, solve the Laplace equation in spherical coordinates with the assumption that the solution is independent of θ. [20%]

4. Solve the following system of linear equations by determining the inverse of the matrix of coefficients,

$$\begin{cases} -x_1 + x_2 = 5 \\ -x_1 + x_3 = -2 \\ 6x_1 - 2x_2 - 3x_3 = 1 \end{cases} \qquad [15\%]$$