所別：大氣物理研究所碩士班 一般生 科目：應用數學

1. Please use (1) the undetermined coefficients; (2) the variation of parameters; and (3) the Laplace transform to solve the following equation,

$$y'' + y = F\cos(t) \quad \text{with} \quad y(0) = A, \ y'(0) = 0.$$

   What is resonance? Can you describe resonance physically with this equation and its associated solution?

   (25%)

2. Define and state the physical meaning of gradient, divergence, and curl. What are Green's, divergence, and Stokes's theorems? Please evaluate the following surface integral,

   $\int_S \mathbf{F} \cdot \mathbf{n} dA$, where $\mathbf{F} = \begin{bmatrix} 0 & 20y & 2z^3 \end{bmatrix}$ and S is the surface of $0 \le x \le 6$,

   $0 \le y \le 1, \ 0 \le z \le y$.

   (20%)

3. For the following matrix,

   $$\mathbf{A} = \begin{bmatrix} 0 & 2 & 0 \\ 3 & -2 & 3 \\ 0 & 3 & 0 \end{bmatrix},$$

   (1) find its determinant; (2) decompose it as the sum of a symmetric matrix $\mathbf{R}$ and a skew-symmetric matrix $\mathbf{S}$; (3) Find the eigenvalues and eigenvectors of $\mathbf{A}$, $\mathbf{R}$, $\mathbf{S}$.

   (20%)

4. (1) Prove that 1, $\sin kx$, and $\cos kx$, with $k = $ integer and $-\pi \le x \le \pi$ form an orthogonal set. (2) Find the Fourier series of the following function,

   $$f(x) = \begin{cases} x & \text{if } -\pi/2 < x < \pi/2 \\ 0 & \text{if } \pi/2 < x < 3\pi/2 \end{cases}$$

   (20%)

5. Solve the following partial differential equation

   $$u_{xx} + 6u_{xy} + 9u_{yy} = 0$$

   (15%)