國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 大氣物理研究所 不分組 科目: 流體力學 共 1 頁 第 1 頁

(15%)1. Define and state the physical meaning of gradient, divergence, vorticity and circulation. What is the relation between vorticity and circulation?

(15%)2. What is body force and what is surface force? Define and state the meaning of normal stress and shearing stresses. State the relations among stresses and velocity in a Newtonian fluid. Write down the Euler's equations of motion and the Navier-Stokes equations.

(10%) 3. A laminar boundary layer velocity profile is approximated by

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^4 \quad \text{for } y \le \delta,$$

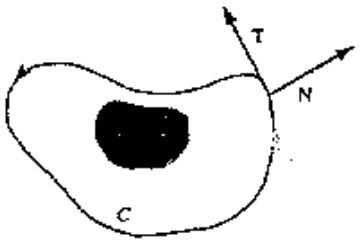
and $u = U$ for $y > \delta$

(a)Show that this profile satisfies the appropriate boundary conditions

(b)Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$.

(15%)4. In a one dimensional flow field the velocity is expressed by u(x,t), and density is $\rho = \rho_0(1 - \sin \omega t)$. If the boundary condition for velocity is u(0,t) = U/2, please find u(x,3) = ?

(10%)5. In the figure shown below, let C denote a regular simple closed curve in the x-y plane, enclosing a



region D. At any point on C, let T be the unit tangential vector in the positive direction and N the unit outer normal vector. Assume velocity vector V is known in region D, except the shaded area. Find a method to compute divergence of V integrated over region D. That is,

$$\iint_D (\nabla \cdot V)\, dxdy \quad = \ ?$$

(10%)6. Let S represent the kinetic energy of a turbulence flow, with the dimension $\left(m^2/s^2\right)$, κ be the wave number, and ε be the kinetic energy dissipation rate. Note that $\varepsilon \propto \frac{u'^3}{l}$

Where u' and l are the turbulence characteristic speed and scale respectively.

If S is only function of κ and ε, that is $S = S(\kappa, \varepsilon)$, please use the method of dimensional analysis to find the relation between S and κ, ε.

(10%) 7. A two-dimensional, incompressible flow field is described by the velocity components

$$\begin{cases} u = Ay \\ v = Bx \end{cases}$$

where A and B are both positive constants. (a) Will the continuity equation be satisfied? (b) Is the flow irrotational? (c) Determine the equation for the streamlines.

(15%) 8. The velocity potential for a certain invisid flow field is

$$\phi = -\left(3x^2y - y^3\right)$$

where ϕ has the units of ft^2/s when x and y are in feet. Determine the pressure difference between the points (1, 2) and (4, 4), where the coordinates are in feet, if the fluid is water and elevation changes are negligible.