所別：太空科學研究所碩士班　　科目：應用數學

1. (a) If $x=r\cos\theta$ and $y=r\sin\theta$, express each the following as a function of r and θ. (10%)

(i) $\left(\frac{\partial\theta}{\partial r}\right)_x$, (ii) $\left(\frac{\partial\theta}{\partial r}\right)_y$.

(b) If $f(x)=\sin x$, where x is in unit of degree, determine expressions for the following: (10%)

(i) $\int f(x)dx$, (ii) Taylor expansion of f(x) about x=0.

2. Find the general solution of the following differential equations:

(a) $(\frac{dy}{dx})^2\frac{d^2y}{dx^2}=1+(\frac{dy}{dx})^2$, (10%)

(b) $(1+2x+x^2)\frac{d^2y}{dx^2}-2y=3x^2+6x+4$. (10%)

3. (a) If $\nabla\bullet r^n\vec{r}=ar^b$, where r is $|\vec{r}|$, $\vec{r}$ is a position vector in spherical coordinate system (r,θ,ϕ). Determine the values of a and b. (10%)

(b) $\vec{F}=\frac{1}{\rho}\hat{e}_\theta$, where ρ,θ is circular cylindrical coordinate system (ρ,θ,z). Determine the values of $\nabla\times\vec{F}$ and $\oint_c\vec{F}\bullet d\vec{r}$, along loop c: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$. (10%)

4. Use residue theorem to evaluate the following integrals:

(a) $\int_{-\infty}^{\infty}\frac{\cos x}{\pi^2-4x^2}dx$, (10%)

(b) $\int_0^{\infty}\frac{x^{m-1}}{x+1}dx$ (0<m<1). (10%)

5. (a) Find a Fourier series of period 6 which in the interval (1, 7) represents a function f(x) taking on the constant value +1 when 1<x<4 and the constant value −1 when 4<x<7. (10%)

(b) Find the value of the following series.(10%)

$\sum_{n_{odd}}\frac{1}{n}\sin\frac{n\pi}{3}=\sin\frac{\pi}{3}+\frac{1}{3}\sin\frac{3\pi}{3}+\frac{1}{5}\sin\frac{5\pi}{3}+\cdots\cdots$.