

國立中央大學八十七學年度碩士班研究生入學試題卷

所別：太空科學研究所 不分組 科目：

應用數學

共一頁 第一頁

1. Solve the following differential equations

[10%] (a) $\frac{dy}{dx} + y \tan x = \sec x$

[10%] (b) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$

[20%] 2. Solve the wave equation

$$\frac{\partial^2 \varphi}{\partial t^2} = c^2 \frac{\partial^2 \varphi}{\partial x^2}$$

subject to the conditions $\varphi(x,0) = F(x)$, $\frac{\partial \varphi(x,0)}{\partial t} = G(x)$

3. Use Residue theorem to evaluate the following integrals

[10%] (a) $\int_0^{2\pi} \frac{d\theta}{A + B \cos \theta}$ ($A^2 > B^2, A < 0$)

[10%] (b) $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ ($a > 0, b > 0$)

4. Given a vector $\vec{A} = \frac{K}{r} \hat{e}_\theta$ (in cylindrical coordinate, where K is a constant)

[8%] (a) Evaluate the $\nabla \times \vec{A}$

[6%] (b) Evaluate the $\oint_C \vec{A} \cdot d\vec{r}$, where C is given by $(x-1)^2 + (y+2)^2 = 16$ and $z = 0$

[6%] (c) Evaluate the $\oint_C \vec{A} \cdot d\vec{r}$, where C is given by $(x-3)^2 + (y+4)^2 = 1$ and $z = 0$

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[10%] (a) Determine the coefficient in the representation

$$f(x) = \sum_{n=1}^{\infty} A_n \sin nx, \text{ where } 0 < x < \pi, \text{ and } f(x) = 1.$$

[10%] (b) Evaluate the value of the following series

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

參考用