所別：太空科學研究所碩士班 一般生 科目：近代物理

1. In the rest frame of an observer receives light from a light source which is moving with a constant velocity $\vec{v}$. The angle between the vector $\vec{v}$ and the line connecting the light source and the observer is θ at the time the light is emitted. If there is no the Doppler effect of the light be observed, what is the angle θ in terms of the speed of $\vec{v}$? (10%)
2. In laboratory frame, a particle of rest mass M and four-momentum P decays into two particles of rest masses m_1 and m_2.
   (a) Find the total energy E_1 of the first particle of mass m_1 in terms of M, m_1, and m_2 in the rest frame of the decaying particle M. (10%)
   (b) Find the kinetic energy T_1 of the first particle of mass m_1 in terms of M, m_1, and m_2 in the rest frame of the decaying particle M. (10%)
   (c) For a special case of the second particle $m_2 = 0$, find the total energy E_2 and the kinetic energy T_2 of the second particle in the same frame. (5%)
3.
   (a) What are the properties of thermal radiation? (5%)
   (b) What is the difference between thermal radiation and blackbody radiation? (5%)
   (c) What are the properties of the spectrum of blackbody radiation that we don't understand based on the theories of the classical thermodynamics and the classical electromagnetism? (5%)
4. A radar system is used for ionosphere measurements at pulses of $3\times10^7\,Hz$.
   (a) What is the minimum uncertain momentum of measurements? (5%)
   (b) What is the minimum uncertain frequency of measurements? (5%)
5. Assume five identical particles of mass m confined in a cubic box of volume V. The particles are free moving within the perfectly rigid and elastic walls of the box.
   (a) If the identical particles are fermions with spin $1/2$.
   Calculate the energy of the box in the ground state. (10%)
   (b) If the identical particles are bosons with spin 0.
   Calculate the energy of the box in the first excited state. (10%)
   (c) Compare the pressure in (a) with the pressure in (b). (5%)
6. The state of wave function of a hydrogen atom is in spherical coordinates (r,θ,ϕ)

$\Psi(r,\theta,\phi)=\frac{1}{8\sqrt{\pi}(a_0)^{3/2}}\frac{r}{a_0}e^{-r/(2a_0)}(\sin\theta)(e^{-i\phi})$, where a_0 is the Bohr radius.

   (a) Find the quantum numbers that label the state of the hydrogen. (10%)
   (b) Find the angular momentum of the electron in the hydrogen atom. (5%)

Note that the useful equations and constants are:

$$\nabla^2 f(r,\theta,\phi)=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r})+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial f}{\partial\theta})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

Planck's constant $h=6.626\times10^{-34}\,J\bullet s$, and $\frac{hc}{e}=1240\,eV\bullet nm$