所別：太空科學研究所碩士班 一般生 科目：近代物理

1. Make two-dimensional space-time diagrams to illustrate and to explain the following concepts,
   (a) the concept of simultaneity. (5%) (b) the time dilation. (5%) (c) the length contraction. (5%)
2. Find the velocity and the momentum of a zero rest mass particle whose kinetic energy is 0.5MeV. (5%)
3. A high speed particle of rest mass m and charge q with total energy E moves at a relativistic velocity in a uniform magnetic field B, perpendicularly. Find the radius of the orbit of the charge particle, in terms of m, q, B, the speed of light c, and the kinetic energy of the particle T. (10%)
4. Let the charges with density ρ_0 in an inertial frame are at rest.
   (a) What is the charge current density 4-vector for another inertial frame with the relative velocity $\vec{u}:(u_x,u_y,u_z)$? (5%)
   (b) Find the scalar invariants associated with the 4-vector current density. (5%)
5. If a radar system is used for distance measurements at pulse widths of 2×10^{-8} sec. What is the uncertain range of measurements? What is the frequency bandwidth of the detecting of the system? (10%)
6. Assuming a particle of mass m confined in an infinitely square potential well of width $-a<x<a$. Calculate the uncertainty product $\Delta x \bullet \Delta p_x$ for the states:
   (a) the ground state. (10%) (b) the first excited state. (15%) (c) the state with energy $E=0$. (5%)
7. The electron in a hydrogen atom is in a state of wavefunction $\Psi(r,\theta,\phi)$ in spherical coordinates (r,θ,ϕ) as $\Psi(r,\theta,\phi)=\frac{-1}{8\sqrt{\pi}(a_0)^{3/2}}(\sin\theta)(e^{i\phi})\frac{r}{a_0}e^{-r/(2a_0)}$, where a_0 is the Bohr radius.
   (a) Calculate the expectation value of the radial distance of the electron. (5%)
   (b) Find the angular momentum of the electron. (15%)

Note the useful equations and constants are:

$$\nabla^2 f(r,\theta,\phi)=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r})+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial f}{\partial\theta})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

The electron mass $m_e=9.11\times10^{-31}kg$, Planck's constant $h=6.626\times10^{-34}J\bullet s$,

$$\frac{hc}{e}=1240eV\bullet nm$$