

1. Explain the following terms.(20%)
(a) Mach cone (b) boundary layer (c) Bernoulli's theorem
(d) stagnation point (e) Joukowski's theorem.
2. (a) Write down the Navier-Stokes equation of an incompressible Newtonian viscous fluid and explain the physical meaning of each term. (6%)
(b) Consider the steady Newtonian viscous fluid flow in a pipe with radius R by a constant pressure gradient G along the pipe (Poiseuille flow in a circular pipe). Determine velocity distribution of the flow in the pipe.(14%)
3. Find the velocity distribution of the uniform ideal fluid flow past a rigid sphere with radius R. (20%)
4. Consider an irrotational flow of the compressible ideal fluid. Assume the main flow is uniform with velocity $\vec{U} = U_0 \hat{i}$, for two dimensional case (x, y), show that $(1-M^2)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ by linearized method, where M is Mach number, ϕ is velocity potential of perturbed velocity. (20%)
5. (a) Use the Navier-Stokes equation of an incompressible Newtonian viscous fluid to derive the vorticity equation $\frac{\partial \vec{\omega}}{\partial t} + (\vec{V} \cdot \nabla) \vec{\omega} = (\vec{\omega} \cdot \nabla) \vec{V} + \nu \nabla^2 \vec{\omega}$, where $\vec{\omega} = \nabla \times \vec{V}$. (14%)
(b) Explain the physical meaning of each term in the vorticity equation. (6%)