國立中央大學九十一學年度碩士班研究生入學試題卷

1. Show that if

$$\frac{dT}{dz} > -\frac{gT}{c_p V}\left(\frac{\partial V}{\partial T}\right)_p,$$

then convection in an atmosphere is absent, where dT/dz is the vertical temperature gradient of the atmosphere, g is the gravitational acceleration, c_p is the specific heat at constant pressure, $V(T,p)$ is specific volume. (15%)

2. Show that the circulation Γ_c around any closed curve C in an inviscid barotropic fluid is constant when following the motion of the fluid. Assume conservative body force. (15%)

3. Integrate the equation of motion for steady inviscid, irrotational flow, with a unique P,ρ-relationship in the presence of conservative body forces. Show that the Bernoulli function for inviscid, irrotational flow is a constant everywhere in the flow region. (Hint: Bernoulli function $\chi = \frac{1}{2}v^2 + V + \int \frac{dp}{\rho}$, where v is flow speed, V is force potential, p is pressure, ρ is mass density.) (15%)

4. Explain the following terms:
   a) streamline, streakline, and pathline, (7%)
   b) vorticity, circulation, and potential flow, (7%)
   c) Reynolds number and Prandtl number, (7%)
   d) Kutta-Joukowski theorem. (7%)

5.
   a) Using scale analysis to derive the two-dimensional Prandtl boundary layer equations. (14%)
   b) Show that the thickness of the boundary layer δ is proportional to $\sqrt{x}$, where x is distance from the leading edge of the plate. (13%)