

所別：水文科學研究所碩士班 科目：應用數學

1. Evaluate the following derivatives:

(a) [5%] $\frac{d(\tan x)}{dx}$ (b) [5%] $\frac{d(e^{x^2})}{dx}$ (c) [5%] $\frac{d(2^x)}{dx}$

2. Evaluate the following integrals:

(a) [5%] $\int_0^{\pi/2a} \cos ax \, dx$ (b) [5%] $\int_0^1 xe^x \, dx$ (c) [5%] $\int_0^2 \int_{-1}^1 (2xy - y^2) \, dx \, dy$

3. [20%] Solve the system of differential equation for $y_1 = y_1(t)$ and $y_2 = y_2(t)$:
$$\begin{cases} y_1' + 3y_1 - y_2 = 0 \\ y_2' - y_1 + 3y_2 = 0 \end{cases}$$

4. [20%] Solve the differential equation for $y = y(x)$: $(3x^2y + y^3)\frac{dy}{dx} + (x^3 + 3xy^2) = 0$.

5. [15%] Represent $\sin ax$ and e^{ax} by Taylor series about $x=0$.

6. [15%] Evaluate $\nabla f \times \nabla f$ and $\nabla(f^2)$ for $f = xy - yz$.

