參考用

(1). 20% As we understand the Hamiltonian expressed in Cartesian frame can be written as

$$\hat{H} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(x,y,z) = \frac{-\hat{p}^2}{2m}+V(x,y,z)$$

we have used the relationship $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ in the above equation. Please evaluate the commentators (a). $[x,p_X]$, (b). $[x,p_X{}^2]$ (c). $[x,p_Y]$, (d).$[x,V(x,y,z)]$ and (e). $[x,H]$.

(2). 10% From the above results we have $[x,H] = \frac{\hbar^2}{m}\frac{\partial}{\partial x} = \frac{i\hbar}{m}\hat{p}_x$. please show that

$$\langle\psi_1|\frac{\partial}{\partial x}|\psi_2\rangle = \frac{m}{\hbar^2}(E_2 - E_1)\langle\psi_1|x|\psi_2\rangle$$

where Ψ_1 and Ψ_2 are eigen functions of Hamiltonian with eigenvalues E_1 and E_2.

(3) 20% As we understand, there existed a common complete sets of eigenfunctions for commuting operators.:

(a). Explain what does it mean by commuting operators.

(b). Define $L_- = L_X - iL_Y$ and $L_+ = L_X + iL_Y$, Please evaluate the commentators $[L_-,L_X]$ and $[L_+, L_Y]$.

(c). Using the above result, show that $L_+L_- = L^2 - L_Z + \hbar L_Z$ and $L_-L_+ = L^2 - L_Z - \hbar L_Z$.

(d). Are the spherical harmonics $Y_{l,m}(\theta, \varphi)$ also eigenfunctions of the operators L_+L_- and L_+L_-?

(e). What are the eigenfunctions expressed in terms of $Y_{l,m}(\theta, \varphi)$ for operators L_+L_- and L_-L_+ with eigenvalues equal to zero?

(4) 7% (a) State the postulates of the kinetic molecular theory of gases.
8%(b) Modify ideal gas equation PV=RT to account for the violation of the postulates for real gas (Van der Waals equation) and explain.

(5) 10% Explain why Li^+ has lower ionic conductivity than Na^+ in aqueous solution and why the value for H^+ is so much higher than the values for both of these ions.

(6) 10% Obtain the rate equation corresponding to the mechanism

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_2} ES' \xrightarrow{k_3} E + Z$$

assuming ES and ES' to be in the steady state and the substrate concentration to be much higher than the enzyme concentration. Express the catalytic constant k_c and the Michaelis constant K_m in terms of k_1, k_{-1}, k_2, and k_3. The rate can be expressed as $rate = k_c[E]_0[S]/(K_m + [S])$

(7) 15% A pressure-volume diagram for the Carnot cycle is shown below: AB and CD are isotherms and BC, DA are adiabatics. Calculate Δ U, q, and w for the four reversible steps and the net change for the whole cycle.