所別：天文研究所碩士班 科目：近代物理

(1) (20 points)

(a) (5 points) From the non-relativistic time dependent Schrödinger equation derive the continuity equation for the probability density $\Psi^*\Psi$. What is the corresponding probability current density (or flux)?

(b) (15 points) A particle is constraint to move in a closed loop of length $L = 1$, but the particle is free to move along the loop. Find the energy levels and the corresponding normalised eigenstates of the particle. State clear the boundary conditions used.
Suppose the initial state of the particle is

$$\Psi(0, x) = \exp(i\, 2\pi x) + \exp(-i\, 4\pi x)\,.$$

What are the probability density and probability current density of the particle at time $t > 0$?

(2) (20 points)

When a particle is in periodic motion, its energy levels may be obtained by the Bohr-Sommerfeld quantization rule

$$\oint p\, dq = nh\,, \qquad n = 1,\, 2,\, \cdots$$

where q is the generalized coordinates and p is the conjugate momentum. The integral is taken over a period.

(a) (10 points) Suppose the proton in an hydrogen atom is infinitely massive and the electron is moving in a circular orbit. Use Bohr-Sommerfeld quantization rule to find the energy levels of the hydrogen atom.

(b) (10 points) A ball is moving vertically under constant gravity. When it contacts with the floor it bounces up elastically. Use Bohr-Sommerfeld quantization rule to find the energy levels of the ball.

(3) (20 points)

(a) (5 points) Describe the differences between fermions and bosons. Name two fermions and two bosons.

(b) (10 points) The spectral energy density (energy density per unit frequency) of black body radiation is

$$u_\nu = \frac{8\pi}{c^3}\,\frac{h\nu^3}{\exp(h\nu/kT) - 1}$$

Derive the low frequency and high frequency approximation of the spectrum.
Show that the frequency corresponding to maximum u_ν is proportional to T.
Derive the energy density u (i.e., integrate u_ν over all ν).
Sketch the spectrum for several temperatures.

(c) (5 points) Describe how classical electromagnetism failed to produce the correct spectrum. Explain how the concept of photon is needed to understand the spectrum.

(4) (20 points)

(a) (10 points) Write down the Lorentz transformation and then derive time dilation and length contraction. Explain the derivation.

(b) (10 points) When a high energy photon (e.g., X-ray) is scattered by a stationary electron, the wavelength of the photon becomes longer. Derive the change in wavelength with respect to the scattering angle.

(5) (20 points)

(a) (5 points) Describe fine structure and hyper-fine structure of hydrogen atom.

(b) (5 points) Describe electronic, vibrational and rotational transitions in molecular spectra.

(c) (5 points) Explain why the life-time of alpha radioactivity is related to the energy of the emitted alpha particle.

(d) (5 points) Name the four forces of nature and the twelve fundamental fermions.