科目：應用數學　　校系所組：中大物理學系、天文研究所　交大電子物理學系丙組、物理研究所
清大物理學系、先進光源學程甲組、天文研究所
陽明生物醫學影像暨放射科學系生物醫學影像組
陽明生醫光電工程研究所自然科學組

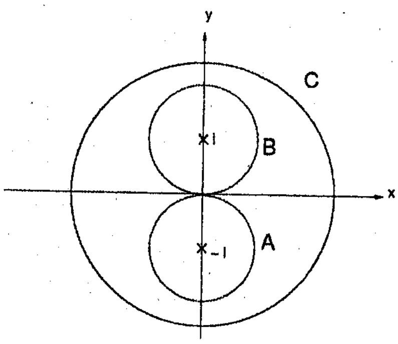
(1) 20 pts

$$f(z) = \frac{1}{z^2+1}$$

is a function of a complex variable $z = x + iy$.

(a). $f(z)$ is analytic except for two simple poles. Find the two poles.

(b). Evaluate the following contour integrals with the contours A, B, and C (going anticlockwise):



(i) $\oint_A f(z)\,dz$

(ii) $\oint_B f(z)\,dz$

(iii) $\oint_C f(z)\,dz$

(2) 25 pts

Let H be a hermitian matrix, i.e., $H^\dagger = H$. If $\mathbf{x_1}$ and $\mathbf{x_2}$ two two eigenvectors of a matrix H with the corresponding eigenvalues λ_1 and λ_2, respectively. That is

$$H\,\mathbf{x_i} = \lambda_i\,\mathbf{x_i}\,, \qquad i = 1,2$$

(i) Show that the eigenvalues are real.

(ii) Show that if λ_1 and λ_2 are different, then $\mathbf{x_1}$ and $\mathbf{x_2}$ are orthogonal to each other.

(iii) Find the two eigenvalues and eignvectors of this matrix

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

*** *Continue on the next page* ***

注意：背面有試題

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(3) 20 pts

$\mathbf{V}(x, y, z)$ is a vector function of (x, y, z) and given by

$$\mathbf{V}(x, y, z) = \hat{\mathbf{x}}(3x + y) + \hat{\mathbf{y}}(z + 2) + \hat{\mathbf{z}}(x^2 - y^2)$$

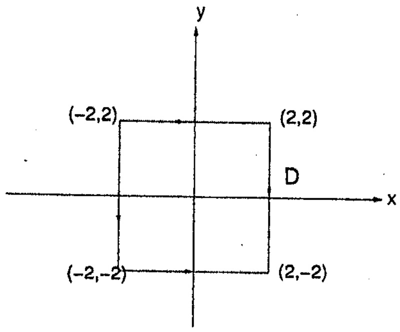
(a). Calculate $\nabla \times \mathbf{V}$

(b). Calculate $\nabla \cdot \mathbf{V}$

(c). Calculate the line integral

$$\oint_C \mathbf{V} \cdot d\lambda$$

along the closed path D shown in the figure.



(4) 20 pts

Consider a second order homogeneous linear differential equation

$$y'' + P(x)\,y' + Q(x)\,y = 0\,.$$

If $y_1(x)$ is a solution to the differential equation, show that the second linearly independent solution can be given by

$$y_2(x) = y_1(x) \int^x \frac{W(x)}{y_1^2(x)}\, dx$$

where $W(x) = \exp[-\int^x P(x)dx]$.

Use this method to find the second independent solution to this equation

$$y'' + \frac{1}{x} y' - \frac{1}{x^2} y = 0$$

given that $y_1(x) = x$.

(5) 15 pts

Denote the Fourier transform of $\frac{d^n f(x)}{dx^n}$ as $g_n(\omega)$, ie.,

$$g_n(\omega) = \int_{-\infty}^{+\infty} \frac{d^n f(x)}{dx^n} e^{i\omega x}\, dx$$

Show that $g_n(\omega) = (-i\omega)^n g_0(\omega)$ if $f(x)$ and derivatives of $f(x)$ vanishes as $x \to \pm\infty$.

**** The End ****

注意：背面有試題