

國立中央大學八十七學年度碩士班研究生入學試題卷

所別：光電科學研究所

不分组 科目：

應用數學

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1. Evaluate the following surface integral by the divergence theorem, assuming that S is oriented as in that theorem :

$$\iint_S (x \, dy \, dz + y \, dz \, dx + z \, dx \, dy), \quad S: x^2 + y^2 + z^2 = a^2 \quad (10\%)$$

2. Show that the following matrix is orthogonal, and determine its eigenvalues :

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (10\%)$$

3. Evaluate

$$I = \int_0^{2\pi} d\phi \frac{b + a \cos \phi}{a^2 + b^2 + 2ab \cos \phi}, \quad |b| > |a|. \quad (10\%)$$

4. Perform Fourier series expansion for the periodic function $f(x)$ with period 2π , where in one period

$$f(x) = |x|, \quad \text{for } |x| < \pi.$$

From the result, find the sum of the following series

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \quad (10\%)$$

5. (a) If $i = \sqrt{-1}$, find the principal value of i^i . (5%)
 (b) Find the general solution for the differential equation

$$y'' + 3y' + 2y = e^x. \quad (5\%)$$

6. Using Laplace transformation method to solve

$$x'' + 2x' + x = t e^{-t}$$

Initial condition: $x_0 = 1, x'_0 = 2$ (10%)

7. Calculate the curve integral

$$\oint_C [(x+y)dx - (x-y)dy],$$

where C is a circular locus of $x^2 + y^2 = r^2$. (10%)

勿用

注意：背面有試題

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8. Solve $y'' + 4y = 4[\sin(2x) + \cos(2x)]$.

Initial condition: $y(\pi) = y'(\pi) = 2\pi$. (10%)

9. Determine the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 7 & 1 & -1 \\ 1 & 4 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

Explain the mathematical meaning of the eigenvalues and eigenvectors compared with the original matrix. (10%)

10. When a circle $x^2 + y^2 = 2x$ in a $z = x + iy$ plane is conformal

mapped into $w = \frac{1}{z}$, find its resultant locus in the $w = u + iv$ plane. (10%)

