國立中央大學九十二學年度碩士班考試入學招生試題卷　　共一頁 第一頁

系所別：物理學系　　科目：近代物理

(1) (25 points)

(a) (5 points) Write down the values (in SI units) of (i) the speed of light, (ii) Planck constant, (iii) the mass of electron, (iv) the mass of proton, and (v) fine structure constant.

(b) (5 points) What is/are the assumption(s) of Einstein's special relativity? Write down the relation between the mass, momentum and energy of a particle in special relativity.

(c) (5 points) Describe an experiment showing the wave nature of electrons, and describe an experiment showing the particle nature of light.

(d) (5 points) What are the difference between bosons and fermions? Give two examples of bosons and two examples of fermions.

(e) (5 points) What is the meaning of degenerate states? Describe Stark and Zeeman effects.

(2) (25 points)

(a) (5 points) How many quantum number does the wavefunction of a hydrogen atom has? What are their ranges?

The radial part of the wavefunction of the 1s-state, 2s-state and 2p-state of an hydrogen atom are given by $R_{1s}(r) = A_{1s}\exp(-r/a_o)$, $R_{2s}(r) = A_{2s}(2 \square\ r/a_o)\exp(\square\ r/2a_o)$ and $R_{2p}(r) = A_{2p}r\exp(\square\ r/2a_o)$, respectively.

(b) (10 points) What is the value of a_o in SI units? Find the normalization constants A_{1s}, A_{2s} and A_{2p}.

(c) (10 points) Find the maxima of the radial probability density of the 1s-state, 2s-state and 2p-state. Sketch the three densities.

(3) (25 points)

(a) (5 points) Write down the energy levels E_n and eigenfunctions $\psi_n(x)$ of a particle in a one-dimensional box $(0 \leq x \leq \pi)$.

(b) (15 points) Calculate the uncertainty in position Δx $(= \sqrt{\langle (x - \langle x \rangle)^2 \rangle})$ and the uncertainty in momentum Δp $(= \sqrt{\langle (p - \langle p \rangle)^2 \rangle})$ of the eigenfunctions in (a). Then verify the uncertainty principle $\Delta x \Delta p \geq \hbar/4$.

(c) (5 points) If the particle's initial state is $\Psi(x,0) = \sin x \cos 3x$, what is its state $\Psi(x,t)$ at a later time t?

(4) (25 points)

(a) (5 points) Describe the Stern-Gerlach experiment and discuss its significance.

The spin of an electron in the direction $\mathbf{A} = \sin\theta\cos\phi\,\hat{e}_x + \sin\theta\sin\phi\,\hat{e}_y + \cos\theta\,\hat{e}_z$, can be represented by $\mathbf{S}_\mathrm{A} = (\sin\theta\cos\phi\,\sigma_x + \sin\theta\sin\phi\,\sigma_y + \cos\theta\,\sigma_z)\hbar/2$, where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & \square i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & \square 1 \end{pmatrix},$$

are the Pauli matrices.

(b) (5 points) Compute the commutators $[\sigma_x, \sigma_y]$, $[\sigma_y, \sigma_z]$, $[\sigma_z, \sigma_x]$.

(c) (10 points) Find the eigenvalues and eigenvectors of $\mathbf{S}_\mathrm{A}$. Thus write down the eigenvalues and eigenvectors of σ_x, σ_y, σ_z.

(d) (5 points) Suppose the electron is spin up in the direction $\hat{e}_z$, find the probability of measuring spin up and spin down in the direction $\mathbf{A}$.