You must show the steps clearly in order to get credits.

Some useful constants: $\epsilon_o = 8.854 \times 10^{-12}$ F/m; $e = 1.6 \times 10^{-19}$ C

(1)[25%] This is a question about the Millikan Oil Drop experiment. An electric field is set up between two horizontal parallel plates, a charged oil drop is allowed to drop through a small hole of the top plate. The drop is first held motionless by the application of the uniform electric field E between the plates. Then the field is switched off and the drop is allowed to fall in air until it reaches the terminal velocity v_T, which is measured in the experiment. The goal of the experiment is to find the charge on the drop, q. Suppose the oil drop is spherical with radius r and density ρ. The viscosity and density of air is η and ρ_A respectively. The gravitational acceleration is g. Use Stoke's Law: the resistant force is $6\pi\eta r v$ for a sphere of radius r moving with speed v in a fluid of viscosity η.

[a] (5%) Use Archimedes' Principle, find the bouyant force due to the air acting on the drop.

[b] (7%)Find the radius of the drop r in terms of η, v_T, ρ and ρ_A.

[c] (8%) Finally express q in terms of all known quantities or directly measured in the experiment.

[d] (5%) What did the oil drop experiment find out about the values of the charge on the oil drops? What is the conclusion from the oil drop experiment about the nature of the electric charge of all matter ?

(2) [20%] A layer of ice of thickness y is on the surface of a lake. The air is at a constant temperature of T and the ice-water interface is at T_0, where $T < T_0$. Given that κ is the thermal conductivity of the ice, L is the latent heat of fusion and ρ is the dendsity of the ice:

[a] (10%) Derive the differential equation for $y(t)$ that governs the rate of growth of the thickness of the ice layer.

[b] (10%) Find the ice layer thickness as a funtion of time , given that the initial thickness at $t = 0$ is y_0.

(3) [15%] A uranium nucleus (charge=92 e, mass=238 u) at rest decays to a thorium nucleus and an α-particle (charge=2 e, mass=4 u). Just after the decay, the particles are at rest and separated by 7.4×10^{-15} m.

[a] (5%) What is the charge and mass of the thorium nucleus ?

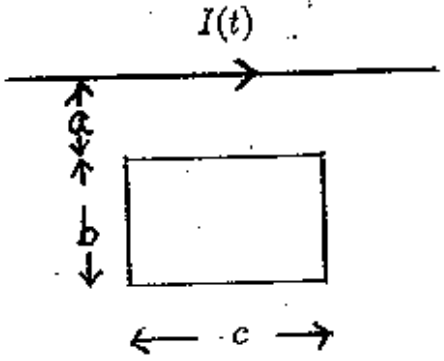
[b] (10%) Find the kinetic energy of each decay particle when they are infinitely far apart. Do no assume that the thorium nucleus stays at rest and do this problem non-relativistically.

(4)[20%] A rectangular loop of size $b \times c$ is placed at a distance a from the wire as shown. The current in the wire varies with time as $I(t) = I_o/t$, where $I_o > 0$. Find

[a] (8%) the magnetic flux through the rectangular loop;

[b] (5%) the direction of the induced emf in the loop;

[c] (7%) the induced emf in the loop.



(5) [20 %] A solid cylinder is released on an incline and rolls without slipping. The moment of inertia of a solid cylinder of radius R and mass M about its symmetric axis is $I = \frac{1}{2}MR^2$.

[a] (10%) Find the acceleration of its center of mass.

[b] (10%) Find the minimum coefficient of friction between the cylinder and the incline needed to prevent slipping.