國立中央大學九十二學年度碩士班考試入學招生試題卷　　共 一 頁 第 一 頁

系所別：物理學系　科目：應用數學

1. (a) [15%] Solve for $y(x)$ from the differential equation: $y'' + y = \frac{1}{\cos x}$.

(b) [10%] Use the Laplace transformation method to solve for $y_1(t)$ and $y_2(t)$ from the coupled differential equations :
$y_1'' - y_1 - 3y_2 = 0$, $y_2'' - 4y_1 + 4e^t = 0$.

2. (a) [15%] Solve the eigenvalue problem for the matrix $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$
Apply the similarity transformation to diagonalize this matrix.

(b) [10%] Find direction of maximum decrease of a position-dependent function $f(x, y, z) = axy + by^3 + cz^2x + d$, at a point $(1, 0, 1)$.
Find also the Laplacian of f there.

3. (a) [15%] Evaluate the Fourier series for a function defined only for $-\pi/2 < x < 3\pi/2$ as : $f(x) = \begin{cases} k, & -\pi/2 < x < \pi/2 \\ 0, & \pi/2 < x < 3\pi/2 \end{cases}$.

(b) [10%] Solve the partial differential equation: $x^2 u_{xy} + 2yu = 0$.

4. (a) [15%] Evaluate the improper integral,

$$\int_0^\infty \frac{\cos ax}{1 + x^2} dx$$

(b) [10%] Find a linear fractional transformation $w = f(z)$ which maps the three points $z_1 = -1, z_2 = 0$ and $z_3 = 1$ onto $w_1 = 0, z_2 = 1$ and $w_3 = -1$ respectively. Try to find critical points of this $f(z)$.