國立中央大學98學年度碩士班考試入學試題卷

所別：統計研究所碩士班 一般生　科目：數理統計　共 2 頁 第 1 頁
學位在職生　＊請在試卷答案卷（卡）內作答

You have to show your work in details for the first 3 problems to get full credit. For the last four, each problem has only one correct answer, but with a penalty of 2 points for wrong answer.

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from the uniform density

$$f(x) = \begin{cases} \frac{1}{\theta}, & 2\theta < x < 3\theta \\ 0, & \text{otherwise} \end{cases},$$

where $\theta > 0$.

(a) Find the maximum likelihood estimator of θ. (10 points)

(b) Find a 90% confidence interval for θ. (10 points)

2. Let $X_1, X_2, \cdots, X_n$ be a random sample from a distribution with density function

$$f(x) = \begin{cases} (1+\theta)x^{\theta}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases},$$

(a) For $n = 1$, in particular, find the most powerful test of size $\alpha = 0.10$ for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$. (10 points)

(b) Based on the random sample $X_1, X_2, \cdots, X_n$, find the form of the best critical region for testing H_0 against H_1. (10 points)

3. Suppose one observation was taken of a random variable X which yielded the value 1/2. The density function of X is

$$g(x \mid \theta) = \begin{cases} \frac{2x}{1-\theta^2}, & \theta \le x \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

and the prior distribution of θ is

$$h(\theta) = \begin{cases} 4\theta(1-\theta^2), & 0 < \theta \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Find the Bayes estimate of θ under the squared error loss. (10 points)

(b) Find a 90% credible set for θ. (10 points)

參考用

注意：背面有試題

4. Let $X_1, X_2, \cdots, X_n, X_{n+1}$ be a random sample from $n(\mu, \sigma^2)$, where μ and σ^2 are both unknown. Set $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$. What is the value of k such that $k(\bar{X}_n - X_{n+1})/S_n$ has a t-distribution?

A. $\frac{1}{\sqrt{n+1}}$　B. $\sqrt{\frac{n}{n+1}}$　C. $\sqrt{\frac{n-1}{n+1}}$　D. $\sqrt{\frac{n(n-1)}{n+1}}$　(10 points)

5. Suppose that $X_1, X_2, \cdots, X_n$ is a random sample from the uniform distribution on (0, 1) and let R be the sample range. What is $P(R \le 1/2)$?

A. $\frac{n}{2^{n-1}}$　B. $\frac{n}{2^n}$　C. $\frac{n+1}{2^n}$　D. $\frac{n+1}{2^{n+1}}$　(10 points)

6. Let X and Y have a bivariate density function which is proportional to xy for $0 < x < y < 1$ and 0, otherwise. What is $E(Y \mid X = x)$ for $0 < x < 1$?

A. $\frac{2(1-x^3)}{3(1-x^2)}$　B. $\frac{2}{3(1-x^2)}$　C. $\frac{2(1-x^3)}{3x^2}$　D. $\frac{2x}{3}$　(10 points)

7. Let $X_1, X_2, \cdots, X_n$ be a random sample of size $n \ge 2$ from a Poisson distribution with mean λ. Consider the following three statistics as estimators of λ.

I. $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$　II. $\frac{1}{n-1}\Sigma(X_i - \bar{X})^2$　III. $2X_1 - X_2$

Which of these statistics are unbiased?

A. I only　B. II only　C. III only　D. I, II and III.　(10 points)

參考用

注意：背面有試題