1. [10 %] Let

$$U = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

be an $n \times n$ matrix. Show that if A is an $n \times n$ matrix such that $UA = A^t$ then $UA = AU$, where A^t denotes the transpose of A.

2. [10 %] Let T be an linear operator on R^4 which is represented in the standard ordered basis by the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$$

Under what conditions on a, b, and c is T diagonalizable?

3. [20 %] Let W be the real vector space spanned by the rows of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 2 & 4 & 1 & 10 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Find a basis for W.

b) Tell which vectors $(x_1, x_2, x_3, x_4, x_5)$ are in W.

c) If $(x_1, x_2, x_3, x_4, x_5)$ is in W, what are its coordinates in the basis chosen in a)?

4. [10 %] Find a projection E which projects R^2 onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

5. [10 %] Suppose that $\{f_n\}$ are a sequence of continuous functions on [0,1] which converge to a function f uniformly on [0,1]. Show that for any $\epsilon > 0$, there is a $\delta > 0$ (δ depends on ϵ only) such that if x, y are any points in [0,1] with $|x-y| < \delta$ then $|f_n(x) - f_n(y)| < \epsilon$, for all n.

6. [10 %] Show that if $\lim_{n\to\infty} a_n = a$ then $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} a_i = a$.

7. [10 %] Let $x, \mu_1, \mu_2 \in R, 0 < \sigma_1, \sigma_2 < \infty$ and $|\rho| \le 1$, evaluate

$$\int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2^2}\right]\right\} dy.$$

(Show your work!)

8. [10 %] Evaluate $\int_0^\infty \frac{\sin \alpha x}{x} dx$ for different values of α.

9. [10 %] Show that for $x > 0$,

$$\frac{x}{1+x^2} e^{-x^2/2} \le \int_x^\infty e^{-y^2/2} dy \le \frac{1}{x} e^{-x^2/2}.$$