國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 統計研究所 不分組 科目: 基礎數學 共 1 頁 第 1 頁

1. Find $\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} \frac{dt}{t+\sqrt{t^2+1}}$. (10%)

2. Find $\int e^{ax}\cos bx\,dx$. (10%)

3. Find $\lim_{n\to\infty} \frac{1}{n}\left\{e^{-\frac{a}{n}} + e^{-\frac{2a}{n}} + \cdots + e^{-\frac{na}{n}}\right\}$. (10%)

4. Let $f(x) = \begin{cases} \frac{g(x)}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$, where g, g'andg'' are continuous at 0 and $g(0) = g'(0) = 0$, $g''(0) = 15$. Find $f'(0)$. (10%)

5. Find $\int_0^1 \frac{3-x}{(x^2-2x+2)^3}dx$. (10%)

6. Let $T : R^2 \to R^3$ be a linear transformation such that $T((x_1, x_2)^t) = (2x_1 - x_2, 3x_1 + 2x_2, x_1 - x_2)^t$

   (1) Find the matrix A associated with T, when R^2 and R^3 are using the standard basis. (2%)

   (2) If $B = \left\{\binom{2}{1}, \binom{1}{2}\right\}$ and $B' = \left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$, find the matrix B associated with T, depending on the bases B and B'. (3%)

   (3) Find matrix Q and P such that A=QBP. (5%)

7. Find the eigenvalues of the matrix A and the associated eigenvectors, where $A = \begin{pmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ \vdots & & \ddots & & \vdots \\ b & \cdots & b & a & b \\ b & \cdots & b & b & a \end{pmatrix}$, $a \neq b$ reals, is a nxn matrix. (10%)

8. Let A be a real symmetric matrix and positive definite. Show that $| A | \leq a_{11}a_{22}\cdots a_{nn}$, where a_{ii} are the ith diagonal element of A, $i = 1, \cdots n$, and $| A |$, the determinant of A. (10%)

9. Let W be a subspace of R^4 spanned by the vectors $(1, 1, 1, 1)^t$ and $(1, 1, -1, 0)^t$. Find the projection of $(1, 2, 3, 4)^t$ onto $W^\perp$. (10%)

10. Let A be a real symmetric matrix, λ_1 the minimum eigenvalue of A, λ_2 the maximum eigenvalue of A. Show that

    (1) $\lambda_1 = \min_{x\neq 0} \frac{x^tAx}{x^tx}$ (5%)

    (2) $\lambda_2 = \max_{x\neq 0} \frac{x^tAx}{x^tx}$ (5%)