國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 統計研究所 組 科目: 基礎數學 共 1 頁 第 1 頁

I. (a) What do you mean by "f is differentiable at x" ? "f is continuous at x" ? (6%)

(b) Show that differentiability implies continuity. Give an example to show that the converse is not true. (9%)

II. Find the Taylor series for each of the following functions about the origin and indicate the interval of convergence. (12%)

(a) $f(x) = e^x$, (b) $f(x) = \cos x$, (c) $f(x) = \ell n\ (1+x)$.

III. (a) What is the Fundamental Theorem of Calculus ? (5%)

(b) Find $\frac{d}{dx}\int_5^{x^2}(t^5+1)^{\frac{1}{2}}dt$. (5%)

IV. Find the point (x, y) on the parabola $x^2 - 3y = 6$ that is closest to the origin. (10%)

V. Using that fact that $\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}x^2}dx = 1$ to evaluate:

(a) $\int_0^{\infty} e^{-x^2}dx$, (4%)

(b) $\int_0^{\infty} x^{-\frac{1}{2}}e^{-x}dx$. (4%)

VI. (a) Prove that $(x'y)^2 \le (x'x)(y'y)$ for all n-vectors x and y. (5%)

(b) If S is an $n \times n$ positive definite matrix, show that $(x'y)^2 \le (x'Sx)(y'S^{-1}y)$ for all n-vectors x, y. (5%)

VII. Let $L : V \to W$ be a linear transformation from V to W. Show that

dim (ker L) + dim (range L) = dim (V),

where ker L $= \{x \in V : L(x) = 0\}$, range $L = \{L(x) : x \in V\}$, and dim$(\cdot)$ denotes the dimension of the vector space $(\cdot)$. (10%)

III. Let A be an $n \times n$ matrix with eigen-values $\lambda_1, \cdots, \lambda_n$. Show that

(a) the determinant of A , $|A| = \prod_{i=1}^{n}\lambda_i$. (4%)

(b) the trace of A, $tr(A) = \sum_{i=1}^{n}\lambda_i$. (4%)

(c) A', the transpose of A , has the same eigen-values as A. (4%)
(Hint: You may consider the characteristic polynomial of A.)

(d) What can you say about the associated eigen-vectors of A' and A. (6%)

IX. Describe the Gram-Schmidt orthonormalization process. (7%)