參考用

Please answer the following questions one by one!

1. Let X_i be independently $N(\mu_x, \sigma^2)$ distributed, for $i = 1, 2, \cdots, n_x$; Y_i be independently $N(\mu_y, \sigma^2)$ distributed, for $i = 1, 2, \cdots, n_y$, such that $\{X_i\}$ and $\{Y_i\}$ be independent. Derive (state reasons) a confidence interval for $\mu_x - \mu_y$ when
(a) σ^2 is known. (5%)
(b) σ^2 is unknown. (10%)
(c) Suppose σ^2 is unknown, derive a test statistic for $H : \mu_x = \mu_y$ against $A : \mu_x \neq \mu_y$ at level α from result (b). (5%)

2. Suppose X has a Poisson distribution with mean λ.
(a) Find the characteristic function of X. (5%)
(b) Define $Z = (X - \lambda)/\sqrt{\lambda}$, show that the limiting distribution of Z as $\lambda \longrightarrow \infty$, is the standard normal distribution. (5%)

3. Let X and Y be two random variables. Show that
(a) $Var(Y) = E[Y - E(Y|X)]^2 + E[E(Y|X) - E(Y)]^2$. (10%)
(b) If $Var(Y) < \infty$, then $Var(Y|X) \leq Var(Y)$. (5%)

4. Suppose $f(x) = \theta^x(1-\theta)^{1-x}$, for x=0,1, and 0 otherwise. Let $H : \theta = 1/10$, $A : \theta > 1/10$. If a sufficiently large sample is taken to justify using the central limit theorem, what critical region of size .05 would you select for this test. (10%)

5. Consider n items whose times to failure $X_1, X_2, \cdots, X_n$ form a sample from the exponential distribution with mean μ. When the experiments stop at time T, $n_1 (0 < n_1 < n)$ failed items are observed, the corresponding failure times are $t_i, i = 1, 2, \cdots, n_1$. And the rest $n - n_1$ items are still alive at time T. Express the likelihood function for this phenomenon. (10%)

6. Just give the answer, don't show your derivations!

(a) Let $X \sim N(0, 1)$, then $Var(X^2)$=__________. (5%)

(b) Given the fact that the expected value of an F variable with n_1 and n_2 degrees of freedom is equal to $n_2/(n_2 - 2)$ for $n_2 > 2$. Then the variance of a t variable with n $(n > 2)$ degrees of freedom is ____ ______ . (5%)

(c) Let $\Psi_X(t) = log\{E[exp(tX)]\}$, for all $|t| < t_0$, where $t_0 > 0$. Then $\Psi'(0)$ = __________ and $\Psi''(0)$ = __________ . (5%)

(d) A population with θ members labeled consecutively from 1 to θ. The population is sampled with replacement and n members of the population are observed and their labels $X_1, X_2, \cdots, X_n$ are recorded. Then ______ is a minimal sufficient statistic for θ. (5%)

(e) Let X_i's be random variables, $E(X_i) = 0$, $Var(X_i) = \sigma^2$, for $i \neq j$, correlation(X_i, X_j) $=\rho$, if $|i - j|$=1, and 0 otherwise. Then $E(\sum_{i=1}^n (X_i - \bar{X})^2)$ = __________, where $\bar{X} = \sum_{i=1}^n X_i/n$. (5%)

(f) Let X_i be i.i.d. Bernoulli(p), $i = 1, 2, \cdots, n$. Find the UMVUE for p. _______ (5%)

(g) Let X_i be i.i.d. $N(0, 1)$ distributed, for i=1,2. Then X_1/X_2 follows __________ distribution. (5%)