科目：工程數學A(5003)　　校系所組：中大照明與顯示科技研究所(乙組)
中大電機工程學系(電子組、固態組)
交大電子研究所(甲組、乙組)
清大電機工程學系（甲組)、光電工程研究所
清大電子工程研究所、工程與系統科學系(丁組)
清大動力機械工程學系(乙組)
陽明醫學工程研究所(醫學電子組)
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參考用

1. The field $\mathbf{Z}_2$ consists of two elements 0 and 1 with the operations of addition (+) and multiplication (·) defined by $0+0=0$, $0+1=1$, $1+0=1$, $1+1=0$, $0\cdot 0=0$, $0\cdot 1=0$, $1\cdot 0=0$, and $1\cdot 1=1$.

Let $A=\begin{pmatrix}1&1&0&0\\0&1&1&1\\0&0&1&0\\1&0&1&1\end{pmatrix}$, $x=\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}$, and $b=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$, where all entries are in $\mathbf{Z}_2$.

(a) (3 %) For the matrix A, compute the rank and the inverse if it exists.
(b) (4 %) Determine whether the system $A\,x=b$ is consistent. If the system is consistent, find all solutions.
(c) (3 %) Find a basis for the solution set of the corresponding homogeneous system.

2. Let $S=\{(1,0,1),(2,2,2)\}$ in $\mathbf{R}^3$, and $\mathbf{W}=\text{span}(S)$.
(a) (6 %) Find an orthonormal basis of $\mathbf{W}$ and $\mathbf{W}^{\perp}$.
(b) (3 %) If $x=(2,0,0)$, find the closet vector u on $\mathbf{W}$ to x.
(c) (3 %) What is the closest distant from $\mathbf{W}$ to x? Please also specify the corresponding vector z.
(d) (3 %) Please plot a schematic diagram that specifies the relation of x, u, z, $\mathbf{W}$ and $\mathbf{W}^{\perp}$.

3. The differential equation: $\ddot{y}(t)+a\dot{y}(t)+by(t)=u(t)$, where a and b are constants and $u(t)$ is the unit step function. All initial conditions are zero.

(a) (5%) Solve $y(t)$ when $a=2$ and $b=4$.
(b) (5%) Solve $y(t)$ when $a=4$ and $b=4$.

4. (5%) EM wave propagates inside an absorptive material. The absorbed intensity amount per penetration depth is proportional to the intensity at that position. Write down a mathematic model to describe the phenomenon above and obtain the general solution.

5. Solve the initial value problems,

(a) (5%) $xy'=y+\sqrt{x^2+y^2}$, $y(2)=0$.

(b) (5%) Solve the initial value problem, $\begin{matrix}y_1'-y_1-y_2=3x\\ y_1'+y_2'-5y_1-2y_2=5\end{matrix}$, with $y_1(0)=3$, $y_2(0)=4$.

6. Evaluate the following integrals.

(a) (6%) $\int_{-\infty}^{\infty}\dfrac{x^3+1}{x^4+1}dx$;

(b) (6%) $\int_{-\infty}^{\infty}\dfrac{\sin(kx)}{x-a}dx$, where $ka=\pi$.

注意：背面有試題

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7. (5%) The following Legendre Equation

$$\left(1-x^2\right)y''-2xy'+\alpha(\alpha+1)y=0$$

with $\alpha=3$ has two linearly independent solutions. Calculate to find the solution that is a polynomial.

8. Let $f(t)$ be a periodic function with period p, and its corresponding Laplace Transform $F(s)=\mathcal{L}\{f(t)\}$ exists.

(a) (4%) Derive the general form of $F(s)$ in terms of a single integral within a finite range. "Finite range" means that neither the upper or lower bound of the integral is infinity.

(b) (4%) Calculate $X(s)$, the Laplace Transform of $x(t)$, from the following initial value problem

$x''+6x'+10x=5f(t)$; $x(0)=x'(0)=0$,

where $f(t)$ is periodic with period 2, and is defined as $f(t)=\delta(t-1)$, for $0\le t<2$, where $\delta(t)$ is the delta function that describes an infinite-sharp impulse.

You **do not** need to perform inverse Laplace Transform to further calculate $x(t)$, so your answer for $X(s)$ should contain a single term but not an infinite series.

9. Consider all piecewise continuous periodic functions $f(t)$ with a period $2L$ that satisfy $f(t)=f(t^-)+f(t^+)$, where $f(t^\pm)$ are the function's right (left) limits at t. We can define the inner product between any two such functions $f_1(t)$ and $f_2(t)$ as

$$f_1\bullet f_1=\int_0^{2L}f_1(t)f_2(t)dt.$$

Moreover, there is a theorem telling that the corresponding Fourier basis functions 1, $\cos\frac{m\pi t}{L}$, and $\sin\frac{m\pi t}{L}$ with m=1, 2, 3, ... form a complete basis set for all such functions, *i.e.*, any such $f(t)$ can be expressed as

$$f(t)=\frac{a_0}{2}+\sum_{m=1}^{\infty}\left(a_m\cos\frac{m\pi t}{L}+b_m\sin\frac{m\pi t}{L}\right)$$

(a) (4%) Using the fact that all the Fourier basis functions are orthogonal to each other, calculate all a_m and b_m.

(b) (8%) Calculate the exact value of **Leibniz's series** $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$

10. Consider the complex function $f(z)=x^2-iy^2$.

(a) (3%) Where is $f(z)$ differentiable?

(b) (3%) Where is $f(z)$ analytic?

11. (7%) Consider a branch $f(z)$ of $(z^2-1)^{1/2}$ that is analytic in the exterior of the unit circle, $|z|>1$. If $f(\sqrt{2})=-1$, find $f(-i\sqrt{2})$.