所別：數學系碩士班 甲組(一般生) 科目：高等微積分
(學位在職生)

1. (20%) Let (M, d) be a metric space and $A, B \subseteq M$. Use $\text{int}(A)$ to denote the interior of A. Prove or disprove that
   (i) $\text{int}(A) \cap \text{int}(B) = \text{int}(A \cap B)$,
   (ii) $\text{int}(A) \cup \text{int}(B) = \text{int}(A \cup B)$.

2. (20%) Let (M, d) be a metric space and $f : M \mapsto M$ satisfy $d(f(x), f(y)) < d(x, y)$ for all $x, y \in M$ with $x \neq y$. Prove that if M is compact, then f has a unique fixed point.

3. (15%) Let $A \subseteq \mathbb{R}$ and $B \subseteq \mathbb{R}$ be bounded below and define $A+B = \{x+y \mid x \in A \text{ and } y \in B\}$. Prove or disprove that $\inf(A + B) = \inf A + \inf B$.

4. (15%) Let K be a compact subset of $\mathbb{R}^n$ and $f : K \mapsto \mathbb{R}$ be continuous. Prove or disprove that $M = \{x \in K \mid f(x) \text{ is the maximum of } f \text{ on } K\}$ is a compact subset of $\mathbb{R}^n$.

5. (15%) Prove or disprove that $f(x) = 1/(x^2 + 1)$ is uniformly continuous on $\mathbb{R}$.

6. (15%) Let $f_n(x) = \frac{nx}{1+nx^2}$, $\frac{1}{\pi} \leq x \leq 2$. Prove or disprove that $\{f_n\}$ converges uniformly on $[\frac{1}{\pi}, 2]$.