國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 高等微積分 共 1 頁 第 1 頁

(17分) 1. Let $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ be a continuous function and $A \subseteq \mathbb{R}^n$ be bounded. Prove or disprove that $f(A)$ is bounded in $\mathbb{R}^m$.

(17分) 2. Prove or disprove that any linear map $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ is continuous.

(17分) 3. Each $f_n : \mathbb{R} \mapsto \mathbb{R}$ is uniformly continuous on $\mathbb{R}$, and $\{f_n\}$ converges uniformly to f on $\mathbb{R}$. Prove or disprove that f is uniformly continuous on $\mathbb{R}$.

(17分) 4 Test the convergence of the series $\sum_{k=1}^{\infty} \frac{\log(k+1)-\log k}{\tan^{-1}(2/k)}$, and explain your reason.

(17分) 5 Let $u(x,y) = \frac{x^4+y^4}{x}$, $v(x,y) = \sin x + \cos y$. Show that the map $(x,y) \mapsto (u,v)$ is locally invertible at $(\frac{\pi}{2}, \frac{\pi}{2})$ and compute $\frac{\partial x}{\partial u}$ at $(x,y) = (\frac{\pi}{2}, \frac{\pi}{2})$.

(15分) 6 Let S denote the upper half surface of ellipsoid $2x^2 + 2y^2 + z^2 = 1$, n denote its outward unit normal vector, and $F(x,y,z) = (9x, 3x + 2x^3 + 6xy^2, 3z)$. Evaluate the surface integral

$$\iint_S \langle \operatorname{curl} F, n \rangle \, dA,$$

where $\langle \cdot, \cdot \rangle$ means the inner product. (此題可直接用定義計算，也可以用Stokes定理求得。)