國立中央大學九十一學年度碩士班研究生入學試題卷

所別：數學系 不分組　科目：線性代數　共 1 頁 第 1 頁

Linear Algebra

(20% each)

1. Let $U = \begin{bmatrix} 1 & 0 & -0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ be the row reduced echelon matrix of the matrix A. Prove or disprove that the first, second and fourth columns of A form a basis for the column space of A.

2. Prove or disprove that

   - $\text{rank}(A \cdot B) \leq \min\{\ \text{rank}(A), \text{rank}(B)\ \}$,
   - $\text{rank}(A \cdot A^T) = \text{rank}(A^T \cdot A) = \text{rank}(A)$.

3. Let $m > n, A$ be a m by n matrix. Prove or disprove that it is always possible to find orthonormal basis $\{V_1, \cdots, V_n\}, \{W_1, \cdots, W_m\}$ for R^n, R^m respectively, such that $A \cdot V_i = c_i \cdot W_i$, $c_i \geq 0$, for $i = 1, 2, \cdots, n$.

4. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find the solution vector of the matrix equation $A \cdot x = b$ with the minimum norm.

5. Prove or disprove that the 4 by 4 matrix $A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix}$ is positive definite.