國立中央大學八十八學年度碩士班研究生入學試題卷

所別：數學研究所 不分組 科目： 線性代數 共 / 頁 第 / 頁

1. Let $A = \begin{bmatrix} 5 & -1 & -1 & -1 \\ -1 & 5 & -1 & -1 \\ -1 & -1 & 5 & -1 \\ -1 & -1 & -1 & 5 \end{bmatrix}$ Find a diagonal matrix D and an orthogonal matrix C such that $C^{-1}AC = D$. (15%)

2. Let A be an $n \times n$ matrix over complex number. Show that the product of all eigenvalues is $\det(A)$. (15%)

3. Let v_1, v_2, v_3, v_4 and v_5 be independent vectors in a vector space V. Let $w_1 = 3v_1 - 2v_2 + v_3$, $w_2 = v_2 + 3v_3 - v_5$ and $w_3 = 2v_1 + v_2 - v_3 + 3v_4 + 2v_5$. Show that w_1, w_2 and w_3 are independent vectors in V. (15%)

4. Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(x,y,z) = (x+y, x-y, y-z)$. Let $\mathbb{B} = ([1,1,-1],[-1,0,1],[0,1,1])$ and $\mathbb{B}' = ([1,0,1],[1,-1,1],[1,-1,0])$ be ordered bases. Denote the matrix representation of T with respect to $\mathbb{B}$ by A and denote the matrix representation of T with respect to $\mathbb{B}'$ by B. Write out A and B. Moreover find an invertible matrix C such that $A = C^{-1}BC$. (15%)

5. Let A be an $m \times n$ matrix. Show that the rank of A^TA equal to the rank of A, where A^T is the transpose of A. (15%)

6. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 4 & 0 & 6 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ if it exists. (10%)

7. Let V be a finite dimensional vector space with $\dim(V) = n$. Let T be a linear transformation from V to itself. Show that for any $v \in V$ there exists an polynomial $p(x)$ such that $\deg(p(x)) \le n$ and $p(T)v = 0$. (15%)