# 國立中央大學八十七學年度碩士班研究生入學試題卷

所別： 數學研究所 不分組 科目： 抽象代數 共 1 頁 第 1 頁

一、Let $G$ be the set of all $2\times 2$ matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $a, b, c, d$ are integers modulo $p$, $p$ a prime number, such that $ad-bc\neq 0$. Prove that $G$ is a non-abelian finite group. (15%).

二、If $N$ is a normal subgroup of a group $G$ and $H$ is any subgroup of $G$, prove that $NH$ is a subgroup of $G$. (15%).

三、Let $G$ be an infinite cyclic group. Prove that the set of automorphisms of $G$ is isomorphic to the cyclic group of order 2. (15%)

四、Describe all groups of order $11^2\cdot 13^2$. (up to isomorphic). (15%).

五、Let $R$ be a commutative ring with unit element. Prove that $R$ is a field if and only if only ideals of $R$ are $(0)$ and $R$. (15%).

六、Prove that $x^2+x+4$ is irreducible over $F$, the field of integers mod 11, and $F[x]/(x^2+x+4)$ is a field having 121 elements. (15%).

七、Prove that it is impossible to duplicate the cube by straightedge and compass. (10%)