國立中央大學八十四學年度碩士班研究生入學試題卷

所別：數學研究所＿＿組＿＿科目：抽象代數　　　　共 1 頁 第 1 頁

1. (20 %) Let G be a group with $|G| = pq$, where $p > q$ are primes and $p \neq 1 \pmod{q}$.

Show that G must be cyclic.

2. (20 %) H is known to be a **maximal normal subgroup** of a group G if $H \neq G$ and for any normal subgroup K of G properly contain H must coincide with G. A **simple group** is one having only two normal subgroups. Let H be a normal subgroup of G,

show that H is a maximal normal subgroup of G if and only if G/H is simple.

3. (20 %) An **integral domain** D is a commutative ring with unity containing no divisors of 0.

Show that finite integral domains are fields.

4. (20 %) A **Euclidean valuation on an integral domain** D is a function μ mapping the nonzero elements of D into nonnegative integers such that the following conditions are satisfied:

i/ for all $a, b \in D$ with $b \neq 0$, there exist q and r in D such that $a = bq + r$, where either $r = 0$ or $\mu(r) < \mu(b)$;

ii/ for all $a, b \in D$, where neither a nor b is 0, $\mu(a) \leq \mu(ab)$.

An integral domain D is a **Euclidean domain** if there exists a Euclidean valuation on D.

Prove that every Euclidean domain is a PID (principal ideal domain).

5. (20 %) Let E be a finite field with characteristic p, P be its prime subfield and n be the degree of E as a finite extension over P.

Show that $|E| = p^n$.

END