國立中央大學八十九學年度碩士班研究生入學試題卷

所別: 數學系 · 不分組　　科目: 離散數學　　共 2 頁 第 1 頁

1. (10%) Let G be a finite graph. Show that G is an induced subgraph of a regular graph.

2. (10%) Show that a tree with n vertices has exactly $n-1$ edges.

3. (15%) $n+1$ integers are chosen from $2n$ distinct integers. Show that among the integers chosen there are two integers a, b such that $\frac{a}{b} = 2^k$ for some positive integer k.

4. (15%) Suppose $d_1, d_2, \cdots, d_n$ are the degrees of all vertices of a graph G. Let $1 \le k \le n$. Show that

$$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{j=k+1}^{n} \min\{k, d_j\}$$

5. (15%) Let l, m be positive integers. Show that every $(lm+1)$-term sequence of real numbers contains either an $(l+1)$-term increasing subsequence or an $(m+1)$-term decreasing subsequence.

6 (15%) Determine the number of words of length n using letters a, b, c such that no two a's appear consecutively.

7. (20%) Solve $h_n = \sum_{k=1}^{n-1} h_k h_{n-k}$ $n = 2, 3, 4, \cdots$

subject to $h_1 = 1$

參考用