國立中央大學八十九學年度碩士班研究生入學試題卷

31 所別: 數學系 不分組 科目: 機率 共 1 頁 第 1 頁

參考用

1. (15分) Two numbers are selected independently at random from the interval $[0,1]$. You are told that the smaller one is less than $\frac{1}{3}$. What is the probability that the larger one is greater than $\frac{3}{4}$?

2. (15分) You are looking for a book in the campus libraries. Each library has it with probability 0.60, but the book may have been borrowed by some other person with probability 0.25. If there are 3 libraries, what are your chances of obtaining this book?

3. (15分) Let X be a discrete random variable with values in $N = \{0, 1, 2 \cdots\}$. Show that $EX = \sum_{n=1}^{\infty} P(X \ge n)$

4. (15分) Let $X_1, X_2, \cdots, X_m$ be m i.i.d. random variables with values in N. Define $r_n = \sum_{k=n}^{\infty} p_k$, where $p_k = P(X_i = k)$, $k \ge 0$. Show that $E[\min(X_1, \cdots, X_m)] = \sum_{n=1}^{\infty} r_n^m$.

5. (10分) Let X_1, X_2 be independent, each with density $f(x) = \frac{1}{\sqrt{2\pi x}} e^{-\frac{x}{2}}$, $x > 0$. Find the density function of $Y_1 = \frac{X_1}{X_1 + X_2}$ and $Y_2 = X_1 + X_2$.

6. (10分) Let X, Y have the joint density function

$$f(x,y) = \begin{cases} x + y & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $\operatorname{Cov}(2X + Y, 3X - Y)$

7. (10分) Let X_1, X_2, X_3 be random sample from the standard normal distribution $N(0,1)$. Find the value δ so that the correlation coefficient ρ of $Y_1 = X_1 + \delta X_3$, $Y_2 = X_2 + \delta X_3$ is $\frac{1}{2}$.

8. (10分) Use the central limit theorem to prove that

$$\lim_{n \to \infty} \sum_{k=0}^{n} \frac{e^{-n} n^k}{k!} = \frac{1}{2}.$$