參考用

(20%) 1. If $u(z)$ is nonconstant and harmonic in the plane, show that $u(z)$ comes arbitrarily close to every real value.

(20%) 2. If $a > 1$, prove that $f(z) = z + e^{-z}$ takes the valuc a at exactly one point in the right half-plane.

(20%) 3. Does there exist a function $f(z)$ analytic in $|z| < 1$ and satisfying

$$f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{2n} \quad (n = 1,\ 2,\ \ldots)?$$

(20%) 4. If $f(z)$ is an entire function and $|f(z)| \le Mr^{\lambda}$ $(|z| = r \ge r_0)$ for some nonnegative real number λ. Prove that $f(z)$ is a polynomial of degree at most λ.

(10%) 5. Compute the integral

$$\int_{|z|=\rho} \frac{|dz|}{|z-a|^2}$$

under the condition $|a| \ne \rho$.

(10%) 6. Determine the LFT $S(z)$ with $S(i) = \infty$, $S(0) = 0$ and S maps $\operatorname{Im} z \ge 0$ onto $\{|w-1| \ge 1\}$.