所別：機械工程學系碩士班 甲、乙組 丙組 科目：工程數學

Ordinary Differential Equation (33%)

1. (a) Please use Laplace Transform to solve the ordinary differential equation of $y'' + 25y = 5\delta(t-\pi)$, with initial conditions of $y(0) = 3$ and $y'(0) = 0$. Note that δ is the Dirac delta function. (12%)

   (b) Calculate the values of $y(\pi/2)$ and $y(2\pi)$. (3%)

2. Solve $y' + y = -2x/y$ with initial condition of $y(0) = 2$. (10%)

3. Solve $y'' - 9y = 0$ with $y(0) = 1$ and $y'(0) = 0$. Please present your answer in the form of Hyperbolic function. (8%)

Linear Algebra & Vector Calculus (33%)

4. For the linear system $\mathbf{Ax} = \mathbf{b}$, where the matrix $\mathbf{A} = [a_{ij}]_{3\times4}$ is given by

$$\mathbf{A} = \begin{bmatrix} -1 & 5 & -1 & -3 \\ 4 & -1 & 2 & 6 \\ 3 & 4 & 1 & 3 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   (a) Find all the possible vectors $\mathbf{b}$ for which the linear system has non-trival solution. (5%)

   (b) Determine the solution $\mathbf{x}$. (5%)

參考用

5. Use Green'theorem to evaluate $\oint_C \vec{F} \cdot d\vec{R}$, where $\vec{F} = xy\vec{i} + xy^2\vec{j}$ and C: the triangle with vertices $(0,0)$, $(3,0)$, $(0,5)$. Note that the curve C is oriented counterclockwise. (8%)

6. Determine the surface area of a sphere of radius a using the technique of surface integral. (8%)

7. Let $\{v_1, v_2\}$ span the vector space of inner product in R^2. Please answer the following questions.

   (a) Is it true that v_1 and v_2 must be mutually orthorgonal? Explain why or why not. (3%)

   (b) Give two examples showing that $\{v_1, v_2\}$ is a orthornormal base in R^2. (4%)

注意：背面有試題

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Fourier Analysis, Partial Differential Equation and Complex Analysis (34%)

8. The function

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \le x < \pi \end{cases}$$

(a) Expand $f(x)$ in a Fourier series. (5%)

(b) Expand $f(x)$ in a complex Fourier series. (5%)

9. (a) Solve the partial differential equation (4%)

$$\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0$$

(b) Solve the boundary-value problem (10%)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \pi, \quad t > 0$$

$$u(0,t) = 0, \quad u(\pi,t) = 0, \quad t > 0$$

$$u(x,0) = \sin x, \quad 0 < x < \pi.$$

10. Using residue calculus, evaluate (10%)

$$I = \int_0^{2\pi} \frac{\cos 2\theta}{5 - 4\sin\theta} d\theta.$$

參考用