

# 國立中央大學九十學年度碩士班研究生入學試題卷

所別: 機械工程學系甲乙丙組 科目: 工程數學 共 1 頁 第 1 頁

1. Solve the following partial differential equation: (25%)

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$$

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = x$$

2. A 16-foot long chain weighting  $m$  pounds per foot hangs over a small pulley, which is 20 feet above the floor. Initially, the chain is held at rest with 7 feet on one side and 9 feet on the other side, as in Figure 1. How long after the chain is released, and with what velocity, will the chain leave the pulley? (25%)

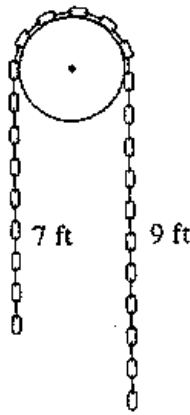


Figure 1  
Chain on  
a pulley.

3. If  $A$  and  $B$  are compatible matrices of rank  $r$  and  $s$ , respectively, prove that  $\text{rank}(AB) \leq \text{minimum}(r, s)$ . (25%)  
(Hint: Using a row partition of  $A$ , show that the rank of  $AB$  is at most equal to  $r$  and then using a column partition of  $B$ , show that the rank of  $AB$  is at most equal to  $s$ .)

參考用

4. Determine the equation of the deflection curve for a cantilever beam  $AB$  subjected to a uniform load of intensity  $q$  (see fig). The D.E of beam deflection is  $EIy'''' = q$  (25%)

