國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 機械工程研究所 (7) 7 万組 科目: 「中、2. 丙工程數學 共 2 頁 第 1 頁

1. Solve the following problems

(1) Let $A = (a_n)$ be an $n \times n$ matrix such that for each i = 1,...n

we have

$$\sum_{i=1}^{n} a_{ij} = 0$$

Show that 0 is an eigenvalue of A. (8%)

(2) A recursive formula is given as

$$r_{n+1} = 4r_n - t_n$$
, $t_{n+1} = 2r_n + t_n$

with the initial values, $r_a = 100$ and $t_a = 10$. Determine

$$\lim_{n\to\infty} \frac{r_n}{t_n} = ? (8\%)$$

(3) A nonhomogeneous system of equations is given as

$$x - 2y + 3z = 1$$

$$2x + ky + 6z = 6$$

$$-x+3y+(k-3)=0$$

Determine the value of k for which (a) the system has a unique solution (3%), (b) the system has no solution (3%), and (c) the system has general solution (3%).



2. Solve the following partial differential equation:

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + 1$$

$$\theta(x,0) = 0$$

$$\frac{\partial \theta(0,t)}{\partial x} = 0$$

$$\theta(L,t)=0$$

Please solve the problem according to the following the steps:

- (1) Separation of variables: $\theta(x, t) = \psi(x, t) + \varphi(x)$. (5%)
- (2) Solve $\varphi(x)$. (5%)
- (3) The variable $\psi(x, t)$ needs a further separation of variable. (5%)
- (4) Solve $\psi(x, t)$ and get the full answer of $\theta(x, t)$. (10%)
- 3. (a) (10%) Find the eigenvalues and eigenvectors of A, where A is defined as

$$\mathbf{A} = \begin{pmatrix} -6 & -4 & -2 \\ -4 & -6 & -2 \\ -2 & -2 & -17 \end{pmatrix}$$

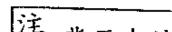
(b) (5%) Find a diagonal matrix D and an orthogonal matrix P, such that

$$\mathbf{D} = \mathbf{P}^T \mathbf{A} \mathbf{P}$$

(c) (10%) Find the solution x(t) for a linear system,

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t)$$

with its initial condition $x(0) = (2,4,3)^T$.



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4. The governing equation for a uniform beam on an elastic foundation

can be written by:
$$\frac{d^4y}{dx^4} + \beta^4 = 0, \text{ where } : \beta^4 = \frac{k}{4EI},$$

y is the deflection of the beam. EI, k are constants.

- (a) Find the general solution of this equation. (10%)
- (b) A semi-infinite beam with concentrated loads, P and M, act at the end x = 0. The deflection y must vanish at $x = \infty$, and the conditions at the origin are:

$$EI\left(\frac{d^2y}{dx^2}\right)_{x=0} = -M, \qquad EI\left(\frac{d^3y}{dx^3}\right)_{x=0} = P,$$

Find the solution of deflection curve y. (15%)

