國立中央大學八十五學年度碩士班研究生入學試題卷

所別：機械工程研究所 甲2. 丙丁組　科目：工程數學　共 2 頁 第 1 頁

一. 弦的一維波動方程式爲：(25%)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

請依序填寫下表中標有號碼之空格（計 5 個），以說明弦振動在不同邊界條件下的運動狀況。 (每小題各佔5%)

〔表一〕

Case	初始條件 I.C.	邊界條件 B.C.	特徵值 eigenvalue	特徵向量 eigenvector	波動方程式全解
I	$u(x,0) = f(x)$ $u_t(x,0) = g(x)$	$u(0,t) = 0$ $u(l,t) = 0$	(1)	(2)	(3)

〔表二〕

Case	初始條件 I.C.	邊界條件 B.C.	波動方程式全解
II	$u(x,0) = f(x)$ $u_t(x,0) = g(x)$	(4)	(a) for x > ct $u(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\tau)d\tau$ (b) for x < ct $u(x,t) = \frac{1}{2}\left[f(x+ct) - f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\tau)d\tau$

〔表三〕

Case	初始條件 I.C.	邊界條件 B.C.	波動方程式全解
III	$u(x,0) = f(x)$ $u_t(x,0) = g(x)$	(5)	$u(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\tau)d\tau$

二. 試解微分方程式系統: (13%)

$$y_1'' = -8y_1 + 2y_2$$
$$y_2'' = 2y_1 - 5y_2$$

其初始條件(initial conditions)爲:
$y_1(0) = 1,\ y_2(0) = -1,\ y_1'(0) = y_2'(0) = 0.$

三. 證明對於所有的三種 Sturm-Liouville Problems 而言, 所有的特徵值(eigenvalues)均爲實數. (12%)

四、(1) For any vectors $\mathbf{F}$ and $\mathbf{G}$, prove that

$$|\mathbf{F}\cdot\mathbf{G}| \le \|\mathbf{F}\|\|\mathbf{G}\| \quad (5\%)$$

(2) Given $\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix}$, determine $\mathbf{A}^{18}$. (5%)

(3) Determine whether the set S of vectors in R^n form a subspace if S consists of all vectors in R^4 with third component 2. (5%)

(4) Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of a given matrix $\mathbf{A}$. Prove that

(a) The inverse $\mathbf{A}^{-1}$ has the eigenvalues $\lambda_1^{-1}, \ldots, \lambda_n^{-1}$, and (5%)

(b) The matrix $\mathbf{A}^m$ has the eigenvalues $\lambda_1^m, \ldots, \lambda_n^m$. (5%)

五.

Let $f(t) = \begin{cases} 0, & 0 \leqq t < \pi \\ 1, & \pi \leqq t < 2\pi, \\ 0, & 2\pi \leqq t \end{cases}$ $y(0) = 0,\ y'(0) = 1.$

(a) Evaluate $\mathcal{L}\{f(t)\}$. (5%)

(b) Solve $y'' + y = f(t)$ by using Laplace transform. (10%)

六.

Let $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \leqq x < \pi \end{cases}$

Expand f(x) in a Fourier Series. (10%)

參考