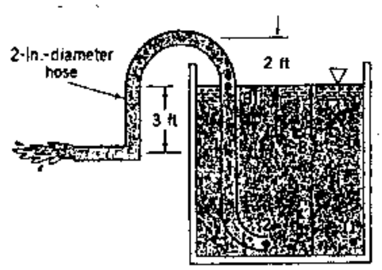
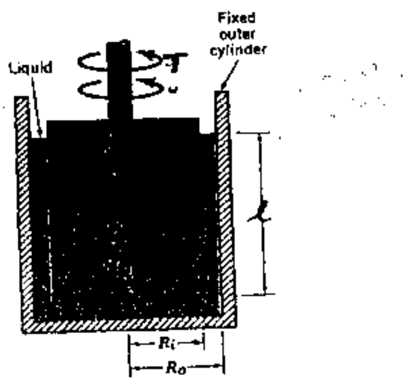
國立中央大學八十五學年度碩士班研究生入學試題卷

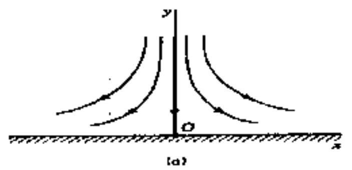
1. (20%) Water is siphoned from the tank. Determine the flowrate from the tank and the pressures at points (1), (2), and (3) if viscous effects are negligible.



2. (20%) Shown in the figure is a rotating cylinder viscometer. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The Torque T required to develop ω is measured and the viscosity μ is calculated from the values of T and ω. Assume the velocity distribution in the gap is linear.
(1) Find the shear stress in the gap in terms of ω, R_i, R_o, μ. (5%)
(2) Find the Torque in terms of $\mu, \omega, \ell, R_i, R_o$. (10%).
(3) Find the appropriate Reynolds number for the flow in the gap. You may use the following terms: ρ(density), $\mu, \omega, \ell, R_i, R_o$. (5%).

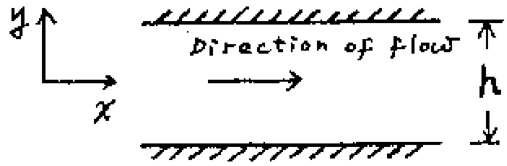


3. (20 %) Potential flow against a flat plate can be described with the stream function $\psi = Axy$, where A is a constant. By adding a source of strength, m, at O, the combined flow becomes a potential flow against a flat plate with a "bump". Determine:
(1) The stream function of the combined flow. (5%)
(2) The velocity component V_r, V_θ. Hint: $V_r = \dfrac{\partial\psi}{r\partial\theta}, V_\theta = -\dfrac{\partial\psi}{\partial r}$ in the cylindrical coordinate. (10%)
(3) The bump height h in terms of A and m. (5%)



4. **Viscous Flow** (20 points)

An incompressible constant-temperature Newtonian fluid flows between the two infinite, horizontal, parallel stationary plates as shown below.



(a) Briefly define the "Newtonian" fluid. (2 pts)

(b) Write down the appropriate assumptions and conditions for the above Couette flow with constant pressure gradient. (3 pts)

(c) Derive the governing equations for the Couette flow in (b) from the incompressible Newtonian equations of fluid motion as given below. (8 pts)

(d) Solve for the velocity profile. (5 pts)

(e) At what value of the appropriate nondimensional pressure gradient does reverse flow (separation) first appear at the fixed surface? (2 pts)

Hint: continuity: $\nabla \cdot \vec{u} = 0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$;

momentum: $\rho\frac{D\vec{u}}{Dt} = -\nabla p + \mu\nabla^2\vec{u} + \rho\vec{f}$; $\vec{f}$ is the body force per unit mass;

more specific, x-direction momentum:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho f_x$$

5. **Compressible Flow** (20 points)

(a) At the seashore, you observe a high-speed aircraft moving overhead at an elevation of 6,000 m. You hear the aircraft 6 s after it passes directly overhead. Estimate the Mach number and the speed of the aircraft. (10pts)

(b) What is f(M) in the relation between differential velocity changes and Mach number changes for isentropic flow of a perfect gas? Simply sketch f(M) vs. M by hand. (10pts)

$$\frac{du}{u} = \frac{dM}{M} f(M)$$

Hint: Since M = u/a, logarithmic differentiation gives $\frac{du}{u} = \frac{da}{a} + \frac{dM}{M}$. The energy equation for isentropic flow is $h + u^2/2 = \text{const}$ and gives $a^2 + \frac{\gamma-1}{2}u^2 = a_0^2$.