

系所別: 機械工程學系 丁組 科目:

工程數學 (含程式設計)

1. Let

$$A = \begin{bmatrix} 1 & 2+i & 3-2i & 4+3i & 5-4i \\ 2-i & 2 & 4-3i & 5+4i & 6-5i \\ 3+2i & 4+3i & 3 & 6+5i & 7-6i \\ 4-3i & 5-4i & 6-5i & 4 & 2 \\ 5+4i & 6+5i & 7+6i & 2 & 5 \end{bmatrix}_{5 \times 5}$$

Prove that all eigenvalues of  $A$  are real. (6%)2. For the linear system of equations  $Ax = b$ , where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & k_1 \\ 3 & k_2 & 0 \\ 4 & 5 & 10 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 1 \\ b_2 \\ 3 \\ 4 \end{bmatrix}$$

- (a) determine the values of  $k_1$ ,  $k_2$ , and  $b_2$ , for which the system has infinitely many solutions; (4%)
- (b) determine the values of  $k_1$ ,  $k_2$ , and  $b_2$ , for which the system has precisely one solution with  $x_3 \neq 0$ ; (4%)
- (c) determine the values of  $k_1$ ,  $k_2$ , and  $b_2$ , for which the system has precisely one solution with  $x_1 = 1$ . (4%)

3. Let  $D = x^{-1}Ax$  be diagonal, with the eigenvalues of  $A$  as the entries on the main diagonal.

(a) Prove that

$$D^m = x^{-1}A^m x \quad (m=2, 3, \dots) \quad (3\%)$$

(b) Find  $A^{10}$  where  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ . (4%)

4.

(a) Solve the problem

$$\frac{dy}{dx} = \frac{-\sin y}{(1+x)\cos y} \quad (4\%)$$

(b) Solve the problem

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0 \quad (5\%)$$

(c) Find the general solution to the equation

$$\frac{d^2 y}{dx^2} + y = \cos x \quad (6\%)$$

(d) What is the relationship between the Fourier transformation and the Laplace transformation?

Can the function  $f(x) = x3^x$  be transformed by the two methods? Explain. (10%)

參考用

注意：背面有試題

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5.

(a) Show that the vector field  $\vec{F} = (x+2y)\vec{i} + (2x-y)\vec{j}$  is a gradient field. Find a potentialfunction for  $\vec{F}$ . Evaluate  $\int_C \vec{F} \cdot d\vec{r}$ ,  $C: (1,0)$  to  $(3,2)$ . (6%)

(b) Evaluate the line integral

$$\oint_C \frac{-y^3 dx + xy^2 dy}{(x^2 + y^2)^2}, \text{ where } C \text{ is the ellipse } x^2 + 4y^2 = 4. \quad (9\%)$$

(c) Use Stokes's theorem to evaluate

$$\oint_C z^2 e^{x^2} dx + xy^2 dy + \tan^{-1} y dz,$$

where  $C$  is the circle  $x^2 + y^2 = 9$ , by finding a surface  $S$  with  $C$  as its boundary and such that the orientation of  $C$  is counterclockwise. (10%)

6. 以下程式問題請使用 Visual Basic 或 C 語法回答。(請註明使用之語法)

(a) 有兩變數  $a$  及  $b$ , 分別為整數及雙倍精度實數, 請宣告. (5%)(b) 有一變數  $D$ , 其值會以等加級數之形式變化, 初始值為 3, 公差為 2, 終值為 93, 請以一迴圈計算. (10%)(c) 有兩不平行向量  $P1(x1,y1,z1)$  及  $P2(x2,y2,z2)$ , 該兩向量會形成一平面, 請寫出一副程式計算平面之單位法向量. (10%)

參考用