## 國立中央大學九十二學年度碩士班考試入學招生試題卷 共 2 頁 第 / 頁

系所別:

機械工程學系 丁組 科目: 工程數學 (含程式設計)

1. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2+i & 3-2i & 4+3i & 5-4i \\ 2-i & 2 & 4-3i & 5+4i & 6-5i \\ 3+2i & 4+3i & 3 & 6+5i & 7-6i \\ 4-3i & 5-4i & 6-5i & 4 & 2 \\ 5+4i & 6+5i & 7+6i & 2 & 5 \end{bmatrix}_{5\times 5}$$

Prove that all eigenvalues of A are real.

2. For the linear system of equations Ax = b, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & k_1 \\ 3 & k_2 & 0 \\ 4 & 5 & 10 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ b_2 \\ 3 \\ 4 \end{bmatrix},$$

- (a) determine the values of  $k_1$ ,  $k_2$ , and  $b_2$ , for which the system has infinitely many solutions;
- (b) determine the values of  $k_1$ ,  $k_2$ , and  $k_2$ , for which the system has precisely one solution with  $x_3 \neq 0$ ;
- (c) determine the values of  $k_1$ ,  $k_2$ , and  $k_2$ , for which the system has precisely one solution with  $x_1 = 1$ .
- 3. Let  $D = x^{-1}Ax$  be <u>diagonal</u>, with the eigenvalues of A as the entries on the main diagonal.
  - (a) Prove that

$$\mathbf{D}^{m} = \mathbf{x}^{-1} \mathbf{A}^{m} \mathbf{x} \qquad (m = 2, 3, ...)$$
 (3%)

(b) Find  $A^{10}$  where  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ . (4%)



(a) Solve the problem

$$\frac{dy}{dx} = \frac{-\sin y}{(1+x)\cos y} \tag{4\%}$$

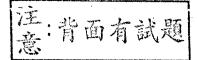
(b) Solve the problem

$$x^{2}\frac{d^{2}y}{dx^{2}} - x\frac{dy}{dx} + y = 0 {(5\%)}$$

(c) Find the general solution to the equation

$$\frac{d^2y}{dx^2} + y = \cos x \tag{6\%}$$

(d) What is the relationship between the Fourier transformation and the Laplace transformation? Can the function  $f(x) = x3^x$  be transformed by the two methods? Explain. (10%)



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5.

- (a) Show that the vector field  $\vec{F} = (x+2y)\vec{i} + (2x-y)\vec{j}$  is a gradient field. Find a potential function for  $\vec{F}$ . Evaluate  $\int_{C} \vec{F} \cdot d\vec{r}$ , C: (1,0) to (3,2).
- (b) Evaluate the line integral

$$\oint_C \frac{-y^3 dx + xy^2 dy}{(x^2 + y^2)^2}, \text{ where } C \text{ is the ellipse } x^2 + 4y^2 = 4.$$
 (9%)

(c) Use Stokes's theorem to evaluate

$$\oint_C z^2 e^{x^2} dx + xy^2 dy + \tan^{-1} y dz,$$

where C is the circle  $x^2 + y^2 = 9$ , by finding a surface S with C as its boundary and such that the orientation of C is counterclockwise. (10%)

- 6. 以下程式問題請使用 Visual Basic 或 C 語法回答. (請註明使用之語法)
  - (a) 有兩變數 a 及 b, 分別爲整數及雙倍精度實數, 請宣告.

(5%)

- (b) 有一變數 D, 其值會以等加級數之形式變化, 初始值爲 3, 公差爲 2, 終值爲 93, 請以一迴圈計算. (10%)
- (c) 有兩不平行向量 P1(x1,y1,z1) 及 P2(x2,y2,z2), 該兩向量會形成一平面, 請寫出一副程式計算平面之單位法向量. (10%)

