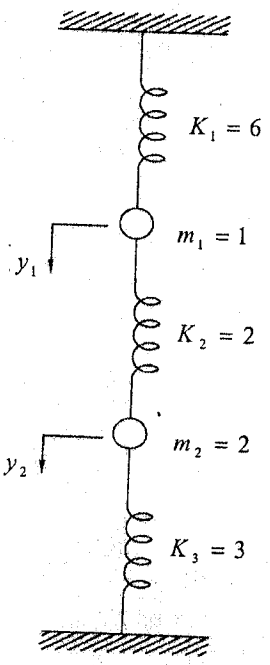
Ordinary Differential Equation (25 %)

1. Consider the mass-spring system as shown in the figure. Assume that there is no damping and that no external force is applied to the system. Suppose that the upper weight is pulled down two units and the lower weight is raised one unit, then both weights are released from rest simultaneously at time $t = 0$.



   (1) Please derive the system of two second order differential equations governing the position of the weights relative to their equilibrium positions at any time $t = 0$. Note that only the system of differential equation is required. (5%)

   (2) Please convert the system of two second order differential equations, you have obtained in (1), into a system of four first order differential equations. (5%)

   (3) Let the system of four first order differential equations be written as $X' = AX$. Determine A and $X(0)$? (5%)

   (4) Determine the eigenvalues of A. (5%)

2. Given

$$J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu}\, m!\, \Gamma(\nu + m + 1)}$$

where $J_\nu(x)$ is known to be the Bessel function of the first kind of order ν, and Γ the Gamma function. Show that

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x. \qquad (5\%).$$

Note that you may directly use $\Gamma(1/2) = \sqrt{\pi}$ without proof.

Linear Algebra & Vector Calculus (25 %)

3. Using Green's theorem, evaluate the line integral $\oint \mathbf{F}(\mathbf{r})d\mathbf{r}$ counterclockwise around the boundary C of a closed region R, where

$\mathbf{F} = [x \cosh(2x), x^2 \sinh(2y)]$, R: $x^2 \le y \le x$. (10%)

4. Evaluate surface integrals $\iint_S \mathbf{F} \cdot \mathbf{n} dA$ for the following data:

$\mathbf{F} = [6x^2, 4y^2, 0]$ S: $\mathbf{r} = [u, v, 3u+6v]$, $1 \le u \le 2$, $-2 \le v \le 2$. (15%)

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Fourier Analysis, Partial Differential Equation and Complex Analysis (25 %)

5. (a) Expand $f(x) = x + \pi, -\pi < x < \pi$ in a Fourier series. (7%)

   (b) Use the result of (a) to find $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ (3%)

6. (a) Solve the wave equation

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, \; t > 0$$

$$u(0, t) = 0, \; u(L, t) = 0, \; t > 0$$

$$u(x, 0) = f(x), \; \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad 0 < x < L \quad (10\%)$$

   (b) Show that the solution of (a) can be written as $u(x,t) = \frac{1}{2}[f(x+at) + f(x-at)]$. (5%)

程式語言 (25 %)

7. 以下使用 c 或 c++語言回答：(10%)

   (a) 宣告兩變數 a 和 b，資料型態分別為雙精度實數與整數。

   (b) 宣告一陣列 c，資料型態為單精度實數，有 20 個元素。

   (c) 以動態記憶體配置方式宣告一陣列 d，資料型態為字元，元素個數設為 num。

8. 有一特徵方程式 $as^2+bs+c=0$，其中 a、b、c 均為實數，擬寫一組程式碼解該方程式，並根據根的型式輸出一旗標 Flag(為整數)。若根為相異實數，Flag=1，若根為重根，Flag=0，其它情況則 Flag=-1。請以函數方式寫出程式碼，輸入為 a、b、c 三參數，輸出為 Flag 與根。(15%)