中央大學八十九學年度碩士班研究生入學試題卷

1. (10%) Determine the curve length from (0,0,0) to (4, 8, 0) for the following vector function:

$$\vec{r}(t) = t\vec{i} + t^{2/3}\vec{j} + 0\vec{k}$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors.

2. (10%) Determine the line integral of the vector function $\vec{F}(\vec{r})$ over a curve C, which is defined by $\vec{r}(t)$. In other words, determine

$$\int_C \vec{F}(\vec{r}) \bullet d\vec{r}$$

where $\vec{F}(\vec{r}) = z\vec{i} + x\vec{j} + y\vec{k}$ and

$$\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + 3t\vec{k} \qquad (0 \le t \le 2\pi)$$

3. (10%) Find the directional derivative of $f(x, y, z) = 2x^2+3y^2+z^2$ at the point (2, 1, 3) in the direction of the vector $(\vec{i} - 2\vec{k})$.

4. Determine the response of the damped-spring system governed by
$y'' + 5y' + 6y = u(t-1) + \delta(t-2)$; $y(0) = 0$, $y'(0) = 1$,
$u(t)$ is the unit step function and $\delta(t)$ is the Dirac delta function. (15%)

5. Find the eigenvalues and eigenfunctions of the following problem.
$x^2y'' + xy' + (\lambda^2x^2 - 4)y = 0$, $y(R) = 0$, $y(0)$ is finite. (10%)

6. Find a general solution of the following problems.
(a) $y'' - 2y' + y = 35x^{3/2}e^x$ (10%)
(b) $x^2y'' - 3xy' + 4y = 0$ (5%)

7. (a) Find the Laplace Transform U(x,s) of the solution of the following boundary value problem: (10%)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \qquad 0 \le x \le 1,\ 0<t$$

B.C. $u(0,t)=u(1,t)=e^t$ for $0<t$

$u(x,0)=1$ for $0<x<1$

(b) One may also solve this problem using the separation of variables. Describe the proper procedures and indicate where you might have difficulties, if there is any. (10%)

8. A generalized one dimension nth order reaction-diffusion equation as following:

$$\frac{\partial w(x,t)}{\partial t} = -kD_{AB}\frac{\partial w}{\partial x} + KC_A^n$$

with the following possible boundary conditions

B.C.:

(i) $w(0,t) = 0,\quad w(L,t)=0$ at any t

(ii) $\frac{\partial w}{\partial t}(0,t) = 0, \frac{\partial w}{\partial t}(L,t) = 0$ at any t

(iii) $\frac{\partial w}{\partial t}(0,t) = 0,\ w(L,t)=0$ at any t

and initial condition:

I.C.:

(i) $w(x,0)=constant$

The questions are (a) What will be the difference of the form of solution (it is not necessary to solve the problem) as L is infinite (∞) or L is limited (5%)

(b) What will be the difference of the solution as the boundary condition (i), (ii) or (iii) with the initial condition (i)? (the form of the solution? sin, cos, or other functions?) (5%)