＊請在試卷答案卷（卡）內作答

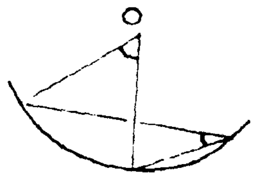
參考用

I.1 試明列推導過程，以驗證

（一） 餘弦函數 $\cos\theta$ 之導數函數 $d\cos\theta/d\theta$ 為 $-\sin\theta$。(10%)

（二） 反正切函數 $\tan^{-1}x$ 之導數函數 $d\tan^{-1}x/dx$ 為 $1/(1+x^2)$。(10%)

I.2 如草圖所示的平面圓弧與圓心 O，試標繪輔助線並列舉角度間關係式，以推得圓心角為兩倍圓周角。(10%)



I.3 就實數對稱方陣 Q 而言，存在固有值(Eigenvalues)與固有向量(Eigenvectors)。

（一） 對任兩個相異的固有值，其所屬的固有向量必然互相垂直，試證明之。(10%)

（二） 倘若對稱方陣能分解為 $Q=C^TC$，於此 C 表示另一方陣；試列式並說明 C 與固有值及固有向量間之關係。(10%)

注意：背面有試題

參考用

II.1 Vectors $v_1, \ldots, v_k$ in R^n are said to be *linearly dependent* if

$$c_1 v_1 + \cdots + c_k v_k = 0 \qquad \text{(II.1-1)}$$

For vectors $v_1 = (3,0,2)$, $v_2 = (2,-1,1)$ and $v_3 = (5,2,4)$, find a condition that v_1, v_2, v_3 are linearly dependent in R^3 . (15%)

II.2 The Fourier transform of a discrete function of one variable, $f(x)$, $x = 0,1,2,\cdots, M-1$ can be described as equation (II.2-1)

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \quad \text{for } u = 0, 1, 2, \ldots, M-1 \qquad \text{(II.2-1)}$$

Similarly, given $F(u)$, the original function $f(x)$ can be obtained with an inverse discrete Fourier transform:

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \quad \text{for } x = 0, 1, 2, \ldots, M-1 \qquad \text{(II.2-2)}$$

Show that $F(u)$ and $f(x)$ are a Fourier transform pair (i.e., substituting $f(x)$ in Eq. II.2-1 with Eq. II.2-2 or substituting Eq. II.2-1 for $F(u)$ into Eq. II.2-2 and the equation holds). (15%)

Note that $\sum_{x=0}^{M-1} e^{j2\pi rx/M} e^{-j2\pi ux/M} = \begin{cases} M & \text{if } r = u \\ 0 & r \neq u \end{cases}$

II.3 The Laplacian operator of a two-variable function $f(x, y)$ is defined as

$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. Show that this Laplacian operator is isotropic (i.e., it is invariant to rotation). (20%)

Note that assume (x', y') is rotating an angle θ of (x, y) then

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

注意：背面有試題