國立中央大學97學年度碩士班考試入學試題卷

所別：工業管理研究所碩士班 甲組　科目：微積分　共 2 頁 第 1 頁

＊請在試卷答案卷（卡）內作答

參考用

1. (10 points) Find the Taylor polynomials (of the indicated degree, and at the indicated point) for the following functions:

   (a) (5 points) $f(x)=x^5+x^3+x$; degree 4, at 0.

   (b) (5 points) $f(x)=\dfrac{1}{1+x^2}$; degree $2n+1$, at 0.

2. (10 points) Many impressive looking limits can be evaluated easily (especially by the person who makes them up), because they are really lower or upper sums in disguise. With this remark as hint, evaluate the following:

$$\lim_{n\to\infty}\left(\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\ldots+\frac{n}{(n+n)^2}\right).$$

3. (10 points) Find f' if $f(x)=[x]$.

4. (10 points) For each of the following functions f, find $f(f'(x))$.

   (a) (5 points) $f(x)=\dfrac{1}{x}$.

   (b) (5 points) $f(x)=17$.

5. (10 points-PROOF) Suppose that $\{f_n\}$ is a sequence of functions which are integrable on [a, b], and that $\{f_n\}$ converges uniformly on [a, b] to a function f which is integrable on [a, b]. Show that

$$\int_a^b f=\lim_{n\to\infty}\int_a^b f_n.$$

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6. (15 points)

Prove that the convergence of $\sum a_n$ implies the convergence of $\sum \frac{\sqrt{a_n}}{n}$, if $a_n \geq 0$.

7. (10 points)

If $\sum a_n$ converges and if $\{b_n\}$ is monotonic and bounded, show that $\sum a_n b_n$ converges

8. (10 points)

Suppose f is defined and differentiable for every $x > 0$, and $f'(x) \to 0$ as $x \to \infty$.

Let $g(x) = f(x+1) - f(x)$. Show that $g(x) \to 0$ as $x \to \infty$.

9. (15 points)

Let E^0 be the set of all interior points of a set E. Show that E^0 is always open.

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