

系所別： 工業管理研究所 甲組 科目：

微積分

1. Let f , g and h be functions satisfying $g(x) \leq f(x) \leq h(x)$ for all $x \in R$, and $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$. Prove that $\lim_{x \rightarrow a} f(x) = L$. (10%)
2. Find the maximum and minimum values of $f(x, y) = xy$ subject to $x^2 + y^2 = 8$. (10%)
3. Show that $x < \tan x$ if $0 < x < \pi/2$. (15%)
4. Show that $|\sin b - \sin a| \leq |b - a|$ for $a < b$. (15%)
5. Find the arc length of the curve $y^4 - 24xy + 48 = 0$ from $(\frac{4}{3}, 2)$ to $(\frac{19}{6}, 4)$. (15%)
6. Evaluate $\int e^x \cos x dx$. (15%)
7. Evaluate $\iint_R e^{xy} dA$, where R is the region enclosed by $y = \frac{x}{2}$, $y = x$, $y = \frac{1}{x}$ and $y = \frac{2}{x}$. (20%)

參考用