1. (20%) Find each of the following infinite sums.

   a) $1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \dfrac{x^4}{4!} - \dots$

   b) $\dfrac{x^2}{2} - \dfrac{x^3}{3\cdot 2} + \dfrac{x^4}{4\cdot 3} - \dfrac{x^5}{5\cdot 4} + \dots$ for $|x| < 1$.

2. (30%) Prove the following theorems.

   a) (15%) If f is differentiable and the graph of f lies above each tangent line except at the point of contact, then f is convex.

   b) (15%) If f is differentiable and f' is increasing, then f is convex.

3. (15%) Find

   a) $\lim\limits_{x\to\infty} a^x$, for $0 < a < 1$

   b) $\lim\limits_{x\to\infty} \dfrac{(\log x)^n}{x}$

   c) $\lim\limits_{x\to 0^+} x^x$

4. (15%) Integration by parts

   a) $\int x^2 e^x \, dx$

   b) $\int e^{ax} \sin bx \, dx$

   c) $\int (\log x)^3 \, dx$

5. (20%) The following figure shows the graph of the *derivative* of f. Find all local maximum and minimum points of f.